

(a) The slope in the  $x$ -direction is:  $\frac{-4 - (-6)}{2 - 1} = 2 = m$ ,  
 while the slope in the  $y$ -direction is:

$\frac{0 - (-6)}{-2 - (-4)} = \frac{6}{2} = 3 = n$ . Since neither of these  
 slopes depends on which  $x$  and  $y$  we choose,  
 the table can represent a linear function.

The function would have to be

$$f(x, y) = -6 + 2(x - 1) + 3(y + 4) \neq \\ = 2x + 3y + 4$$

(b) Although, for fixed  $x$ , the slope with respect  
 to  $y$  is constant and, for fixed  $y$ , the slope  
 with respect to  $x$  is constant, the slope  
 with respect to  $x$  varies across the different  
 columns. For example, the slope for

$$y = -4 \text{ is } m = \frac{-10 - (-7)}{2 - 1} = \frac{-3}{1} = -3$$

while the slope for  $y = -2$  is

$$m = \frac{-4 - (-3)}{2 - 1} = -1 \neq -3.$$

Therefore, the table cannot represent a  
 linear function.

② The contours are parallel and equally spaced, so they can represent a linear function.

Examining the  $-4$  contour and  $2$  contour at  $x=2$ , we see that the slope in the  $y$  ~~direction~~ direction must be:  $n_{\cancel{y}} = \frac{2 - (-4)}{1 - (-1)} = \frac{6}{2} = \boxed{3}$

Examining the  $4$  contour and  $2$  contour at  $y=-1$  gives that the slope in the  $x$ -direction must be  $m = \frac{2-4}{-1-(-2)} = \frac{-2}{1} = \boxed{-2}$ .

Since  $f(0, -1) = 0$ , we obtain:

$$\begin{aligned} f(x, y) &= 0 - 2x + 3(y - (-1)) \\ &= -2x + 3(y + 1) \\ &= -2x + 3y + 3 \end{aligned}$$

- (3) (a) The distance of  $(x, y, z)$  from the  $xy$ -plane is  $|z|$ , while the distance of  $(x, y, z)$  from the  $z$ -axis is  $\sqrt{x^2 + y^2}$ .

We thus have

$$|z| = (\sqrt{x^2 + y^2})^2 = x^2 + y^2.$$

That is,  $z = \pm (x^2 + y^2)$

- (b) The graph is two paraboloids, one opening down and one opening up.

(c)

