① $f(x,y) = x^2 \sin(x+y)$, so $\frac{\partial f}{\partial x} = 2x\sin(x+y) + x^2\cos(x+y)$.

$$\frac{\partial f}{\partial y} = x^2\cos(x+y).$$

$$\frac{\partial^2 f}{\partial x^2} = 2\sin(x+y) + 2x\cos(x+y) + 2x\cos(x+y) - x^2\sin(x+y)$$

$$= 2\sin(x+y) + 4x\cos(x+y) - x^2\sin(x+y)$$

$$\frac{\partial^2 f}{\partial y^2} = -x^2\sin(x+y)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(x^2\cos(x+y)\right) = \frac{\partial}{\partial y}\left(2x\sin(x+y) + x^2\cos(x+y)\right)$$

$$= 2x\cos(x+y) - x^2\sin(x+y)$$

---

② $\nabla f = \begin{bmatrix} 2x\sin(x+y) + x^2\cos(x+y) \\ x^2\cos(x+y) \end{bmatrix}$,

$$\nabla f(\pi/2, \pi/2) = \begin{bmatrix} 2(\pi/2)\sin(\pi) + (\pi/2)^2\cos(\pi) \\ (\pi/2)^2\cos(\pi) \end{bmatrix} = \begin{bmatrix} -\pi^2/4 \\ -\pi^2/4 \end{bmatrix}.$$

$$f_{\vec{u}} = \nabla f \circ \vec{u} = \begin{bmatrix} -\pi^2/4 \\ -\pi^2/4 \end{bmatrix} \circ \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = -\pi^2/4\left(\frac{1}{\sqrt{2}}\right) + \left(-\pi^2/4\right)\left(\frac{-1}{\sqrt{2}}\right) = \boxed{0}$$

(3) (a) $H(5,10,6) = .5e^{-\frac{(25)/500}{.100}} = (.5e^{-.05})\cdot 100$

(b) The mosquito will fly in the direction of ∇H.

$$\nabla H: \quad \frac{\partial H}{\partial x} = 100\left(\frac{(z-6)^2}{72}+.5\right)\left(\frac{-x}{250}\right)e^{-\left(\frac{x^2+(y-10)^2}{500}\right)}$$

$$\frac{\partial H}{\partial y} = 100\left(\frac{(z-6)^2}{72}+.5\right)\left(\frac{-(y-10)}{250}\right)e^{-\left(x^2+(y-10)^2\right)/500}$$

$$\frac{\partial H}{\partial z} = 100\left(\frac{z-6}{36}\right)e^{-(x^2+(y-10)^2)/500}$$

Evaluating at (5,10,6) gives

$$\nabla H = 100\begin{bmatrix} -.5/25 \\ 0 \\ 0 \end{bmatrix}e^{-.05}.$$ Thus, the mosquito will fly in the negative x-direction.