1. A direction vector is $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Thus, a parametrization for the line is:

$$\vec{r}(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+t \\ 1+2t \\ 1+3t \end{bmatrix}.$$

That is, $x(t) = 1+t$

$y(t) = 1+2t$

$z(t) = 1+3t$.

Note: This is not the only correct parametrization.

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2. Direction vectors for the plane are:

$$\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \text{ and } \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Thus, one possible parametrization for the plane is:

$$\vec{r}(s,t) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$

that is, $x(s,t) = 1-s-t$

$y(s,t) = s$

$z(s,t) = t$

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3 A parametrization of $\mathcal{C}$ is: $x(t)=t,\ y(t)=t^2,\ 0 \le t \le 1.$

Thus,

$$\int_{\mathcal{C}} \vec{F} \circ d\vec{r} = \int_{t=0}^{1} \begin{bmatrix} t^2 \\ -t \end{bmatrix} \circ \begin{bmatrix} 1 \\ 2t \end{bmatrix} dt = \int_{t=0}^{1} t^2 - 2t^2\, dt$$

$$= -\frac{1}{3} t^3 \Big|_{t=0}^{1} = \boxed{\frac{-1}{3}}.$$

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4 If $\vec{F} = \nabla f$, and $\vec{F} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$, then $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$.

However, $\frac{\partial f_1}{\partial y} = 1$, while $\frac{\partial f_2}{\partial x} = -1$. Therefore, the vector field is not conservative.