Math. 302-03
Spring, 2008
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Final Examination

Tuesday, May 13, 2008, 08:00–10:30

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this copy of the exam questions. Each entire problem is worth 12 points, and 4 points are "free."

1. A person swimming in the ocean, 0.5 miles south of the beach, encounters a cross-current of 1 mile per hour flowing west, parallel to the beach. How fast and in what direction would the person need to swim to follow a line back to the beach that is perpendicular to the beach, such that the person gets back to the beach in an hour? Give the direction the person must swim as an angle with respect to east. *Hint: It may help to draw a picture.*

2. Suppose the salt concentration in percent in the ocean x miles south and y miles west of a certain city is
$$f(x,y) = 0.03 * \left(1 - e^{-(4x+y)}\right).$$
Suppose also that a certain kind of fish will swim in the direction in which the salt concentration increases most rapidly. In what direction would that fish swim if it finds itself at the point $(0,1)$? Give the direction as a unit vector (u_1, u_2), and draw the direction on a diagram.

3. Find the global maximum and all global maximizers and minimizers of the function $f(x,y) = x + y^2$ over the disk $x^2 + y^2 \leq 4$.

4. Write down parametric equations for:
   (a) the line through $(1,2,3)$ and $(5,5,5)$;
   (b) the circle of radius 2 centered at the point $(3,4)$.

5. Write down a parametrization of the sphere of radius 1 centered at $(2,3,4)$.

6. Compute the line integral of $\vec{F}(x,y,z) = (y,y,z)$ over the portion of the helix $\vec{r}(t) = (\cos(t), \sin(t), t)$ defined by $0 \leq t \leq \pi$.

7. Use Green's theorem to evaluate the line integral of $\vec{F}(x,y) = (y,y)$ over the circle of radius 3 centered at $(10^6, 10^7)$.

8. Compute the integral of $f(x,y) = xy$ over the area enclosed by the curves $y = x^3$ and $y = x^2$.