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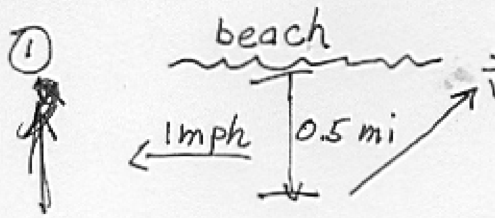
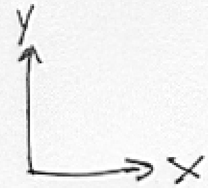
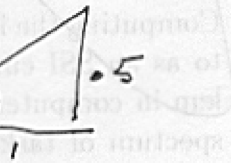
(1)



The current vector is $(-1,0)$, and the swimmer's velocity vector must obey $\vec{v} + (-1,0) = (0,a)$.



Now, a must have $1-a=0.5$. Thus, $v_1 - 1 = 0 \Rightarrow v_1 = 1, v_2 = 0.5$, so $\vec{v} = (1, 0.5)$. The angle is thus $\arctan(.5) = \theta \approx .4636$ radians $\approx 26.5^\circ$,



and the swimmer's speed should be $\sqrt{1^2 + .5^2} \approx 1.12$ mph

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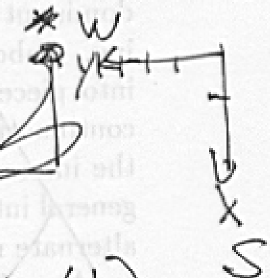
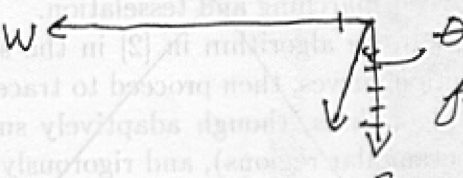
(2) The fish would swim in the direction of $\nabla f = \left(0.03(4e^{-(4x+y)}), 0.03(e^{-(4x+y)})\right) = 0.03e^{-(4x+y)}(4,1)$.

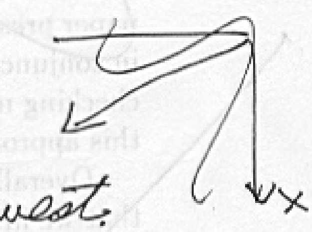


A unit vector in this direction is $\left(\frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}}\right)$. The angle would be $\theta = \tan^{-1}\left(\frac{1}{4}\right)$

$\theta \approx 0.245$ radians $\approx 14^\circ$. It would go



4 miles south for every mile west.



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(3) (a) The critical points inside the disk would have to obey: $\nabla f = \begin{bmatrix} 1 \\ 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; there are no such points.

(b) We may use Lagrange multipliers to find the critical points on the boundary of the disk:

$$\nabla f = \lambda \nabla g, \text{ where } \nabla g = \nabla(x^2+y^2-4) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}.$$

(continued on next page)

We have $\begin{bmatrix}1\\2y\end{bmatrix} = \lambda\begin{bmatrix}2x\\2y\end{bmatrix}$, i.e. $1 = 2x\lambda$ and $2y = \lambda(2y)$.

The first equation gives $x \neq 0$, $\lambda \neq 0$, and the second equation gives ~~x=0 or~~ $y = 0$ or $\lambda = 1$.

If $y=0$, then $x = \pm 2$. If $\lambda = 1$, then $x = \frac{1}{2}$, $y = \pm\sqrt{4 - 1/4} = \pm\frac{\sqrt{15}}{2}$

The candidates are thus:

x	y	$f(x,y)$
-2	0	-2
2	0	2
$1/2$	$-\frac{\sqrt{15}}{2}$	$17/4$
$1/2$	$\sqrt{15}/2$	$17/4$

The minimum is thus $f = -2$, and it occurs only at $(x,y) = (-2,0)$.

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(4) (a) A direction vector is $(5,5,5)-(1,2,3) = (4,3,2)$.

Parametric equations are thus:

$\vec{r}(t) = \begin{bmatrix}1\\2\\3\end{bmatrix} + t\begin{bmatrix}4\\3\\2\end{bmatrix}$, that is: $x(t) = 1+4t$, $y(t) = 2+3t$, $z(t) = 3+2t$

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(5) A sphere of radius 1 centered at $(2,3,4)$ has parametric representation:

$x = \sin v \cos u$, $0 \le u \le 2\pi$, $0 \le v \le \pi$.

$y = \sin v \sin u$

$z = \cos v$

A sphere of radius 1 centered at $(2,3,4)$ thus has parametric representation:

$x = 2 + \sin v \cos u$

$y = 3 + \sin v \sin u$

$z = 4 + \cos v$.

(6) $$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \int_{t=0}^{\pi} (\sin(t),\sin(t),t)\circ(-\sin(t),\cos(t),1)\,dt$$

$$= \int_{t=0}^{\pi} (-\sin^2 t + \cos t \sin t + t)\,dt = \int_{t=0}^{\pi} -\frac{1-\cos(2t)}{2} + \frac{1}{2}\overset{\sin}{\cancel{\cos}}(2t) + t\,dt$$

$$= \int_{t=0}^{\pi} \cancel{\cos(2t)} - \frac{1}{2} + t\,dt = \left[-\frac{t}{2} + \frac{t^2}{2}\right]\Big|_{t=0}^{\pi} = \boxed{\frac{\pi^2-\pi}{2}}$$

These integrate to 0 over $[0,\pi]$.

(7) $$\int_{\mathcal{C}} \vec{F}\circ dr = \iint_R \mathrm{curl}(F)\,dA = \iint_R -1\,dA = -\left[\begin{array}{l}\text{area of circle}\\ \text{of radius } 3\end{array}\right]$$

$$= \boxed{-9\pi}$$

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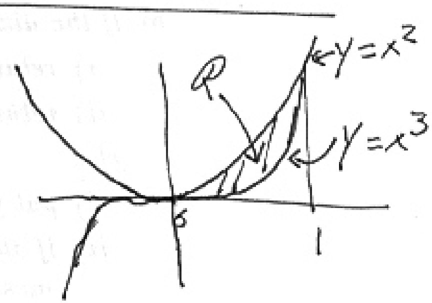
(8) $$\iint_R xy\,dA = \int_{x=0}^{1} x \int_{y=x^3}^{x^2} y\,dy\,dx$$



$$= \int_{x=0}^{1} x\,\frac{y^2}{2}\Big|_{y=x^3}^{x^2} dx = \frac{1}{2}\int_{x=0}^{1} x\left[x^4 - x^6\right]dx$$

$$= \frac{1}{2}\int_{x=0}^{1} x^7 - x^5\,dx = \frac{1}{2}\left[\frac{x^8}{8} - \frac{x^6}{6}\right]\Big|_{x=0}^{1} = \frac{1}{2}\left[\frac{1}{6} - \frac{1}{8}\right] = \frac{1}{2}\left[\frac{4}{24} - \frac{3}{24}\right]$$

$$= \frac{1}{2}\left[\frac{1}{24}\right] = \boxed{\frac{1}{48}}$$