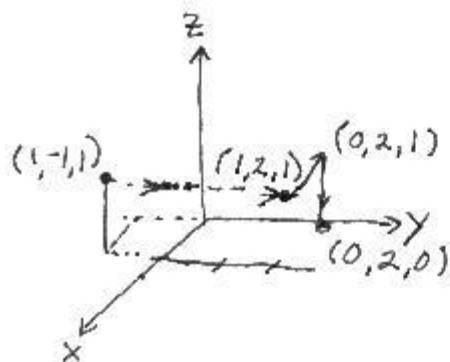


①



You will end up at  $(0, 2, 0)$ .

②  $z = mx + ny + c$   $c = -3, m = 4, n = -5.$

$\therefore z = 4x - 5y - 3$

③ Assuming the plane is not vertical, its equation is  $z = mx + ny + c$ . (The general equation can be  $ax + by + cz = 1$ .)

Plugging in gives:

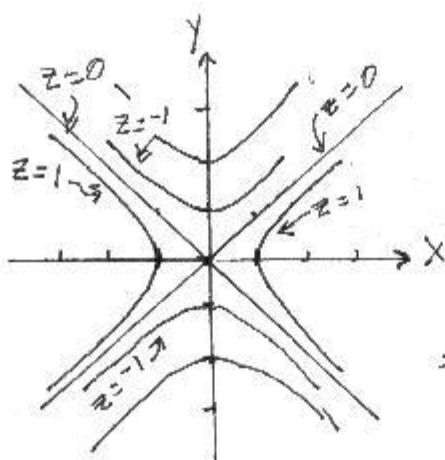
$(0, 0, 1): 1 = m \cdot 0 + n \cdot 0 + c, \text{ i.e. } c = 1.$

$(1, 0, 2): 2 = m + 1 \Rightarrow m = 1.$

$(1, 1, 3): 3 = 1 + 1 + n \Rightarrow n = 1.$

$\therefore z = x + y + 1$

④



$z = 0: x^2 - y^2 = 0$

$z = 1: x^2 = y^2 + 1$

$z = -1: y^2 = x^2 + 1$

For  $z = 0$ , the contour is a set of intersecting lines. For all other values of  $z$ , the contours are hyperbolas.

- (5)  $f(x,y) = \frac{x^2 - y^2}{x+y} = x-y$  for  $x+y \neq 0$ . Therefore, the function has a limit of  $0 = 0+0$  at  $(0,0)$ , provided we exclude the line  $y = -x$  from its domain.

- (6) If  ~~$y = x$~~   $y = 2x$ :  $f(x, 2x) = \frac{x^2 + 4x^2}{x^2 - 4x^2} = \frac{5x^2}{-3x^2} = -\frac{5}{3}$ .  
If  $2y = x$ :  $f(2y, y) = \frac{4y^2 + y^2}{4y^2 - y^2} = \frac{5}{3} \neq -\frac{5}{3}$ .  
Therefore,  $f$  cannot have a limit at  $(0,0)$ .

- (7) The function can be any function of  $y^2 + z^2$ . For example,  $f(x,y,z) = (y^2 + z^2)^2$  would do.