Math 302-03, Second examination, answers, p. 1 2008/02/~~28~~27

1. Vectors in the plane are $\vec{v}^{(1)}=(1,0,2)-(0,0,1)=(1,0,1)$ and $(1,1,3)-(0,0,1)=(1,1,2)$. A normal to the plane is thus given by:

$$\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 1\\ 1 & 1 & 2\end{vmatrix}=\vec{i}\begin{vmatrix}0 & 1\\ 1 & 2\end{vmatrix}-\vec{j}\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}+\vec{k}\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}$$

$$=-\vec{i}-\vec{j}\,(+1)+\vec{k}=(-1,1,1).$$

An equation for the plane is thus:

$$-(x-0)-(y-0)+(z-1)=0.$$

Simplifying gives:

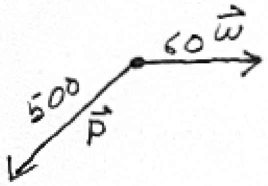
$$\boxed{-x-y+z=1}$$

2. A normal to the plane is $(1,1,1)$. An equation for the plane is thus:

$$1\cdot(x-2)+1\cdot(y-3)+1\cdot(z-4)=0,$$

or, simplifying, $\boxed{x+y+z=9}$.

3.



The ground ~~speed~~ vector is $\vec{g}=\vec{p}+\vec{w}$. $\vec{p}=(500\cos 5\pi/4,\ 500\sin 5\pi/4)$

$$=(-250\sqrt{2},\,-250\sqrt{2}).$$

$$\vec{w}=(60,0).$$

Thus, $\vec{g}=(60-250\sqrt{2},\,-250\sqrt{2})$. The ground speed is $\sqrt{(60-250\sqrt{2})^2+(-250\sqrt{2})^2}\approx 459.5$ miles per hour.

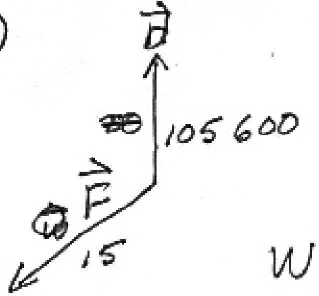
The angle θ of $\vec{g}$ obeys $\tan\theta=\dfrac{-250\sqrt{2}}{60-250\sqrt{2}}$

Since θ is a third-quadrant angle,

$$\theta\approx 4.0194 \text{ radians} \approx 4.0194\left(\frac{180}{\pi}\right)\text{ degrees}$$

$$\approx 230^\circ \approx 180+50^\circ.$$

4



$W = \vec{F} \circ \vec{d}$ $\quad \vec{d}\,\cancel{\vec{F}} = (0, 105600)$

$\vec{F}\,\cancel{\vec{d}} = (15\cos 7\pi/6,\ 15\sin 7\pi/6).$

$W = 0\,(15\cos 7\pi/6) + 105600\,(15\sin 7\pi/6)$

$= (105600)(15)\left(\frac{-1}{2}\right) = -792{,}000 \text{ ft. lbs.}$