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1. (a) $$\frac{\partial f}{\partial x} = e^{-xy} - xye^{-xy} = (1-xy)e^{-xy}.$$

(b) $$\frac{\partial f}{\partial y} = -x^2e^{-xy}$$

(c) $$\frac{\partial^2 f}{\partial x^2} = \cancel{-e^{-xy}} - ye^{-xy} - y(1-xy)e^{-xy} = -y(2-xy)e^{-xy}$$

(d) $$\frac{\partial^2 f}{\partial y^2} = x^3e^{-xy}.$$

(e) $$\frac{\partial^2 f}{\partial x \partial y} = -2xe^{-xy} + x^2ye^{-xy} = x(-2+xy)e^{-xy}$$ ✓

Check:

$$\frac{\partial^2 f}{\partial y \partial x} = -xe^{-xy} - x(1-xy)e^{-xy} = x(-2+xy)e^{-xy}$$

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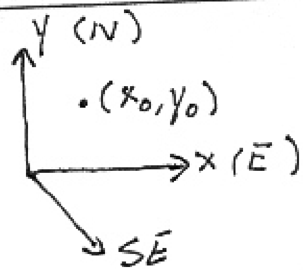
2. 

$$f(x_0, y_0) = 7135$$

$$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{1200}{5000} = 0.24$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{2500}{5000} = 0.5.$$

Southeast corresponds to a direction vector of $\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \vec{u}$.

Thus, $D_u f(x_0, y_0) = \nabla f(x_0, y_0) \circ \vec{u} = 0.24\left(\frac{1}{\sqrt{2}}\right) + 0.5\left(-1/\sqrt{2}\right)$

$= -0.26/\sqrt{2}$.

If you go 100 ft. in that direction, then you will descend by about $100(.26)/\sqrt{2} \approx 18.4$ ft.

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3. $\nabla A = \begin{bmatrix} 2x \\ -4y \end{bmatrix}$ $\dfrac{\partial A(x(t), y(t))}{\partial t} = \nabla A(x(t), y(t)) \circ \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$

$$= 2x(2t) + (-4y)(3t^2)$$

$$= 2(t^2-1)(2t) + (-4t^3)(3t^2)$$

$$= 2(4-1)(4) + (-32)(12)$$

$$= 24 - 384 = \boxed{-360}$$

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4. $f(1,0) = 1$. $\dfrac{\partial f}{\partial x}(1,0) = 1$; $\dfrac{\partial f}{\partial y}(1,0) = -1$.

$$\boxed{z - 1 = (x-1) - (y-0)}$$