Math. 302-03, 2008/04/08 – Fourth exam answers – page ①

① The critical points are where $\nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$$\frac{\partial f}{\partial x} = 2x - y - 3 \; ; \; \frac{\partial f}{\partial y} = 2y - x$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow x = 2y, \text{ and } \frac{\partial f}{\partial x} = 0 \Rightarrow y = 2x - 3.$$

Plugging in gives $y = 4y - 3 \Leftrightarrow -3y = -3 \Leftrightarrow y = 1 \Rightarrow x = 2.$

$\therefore$ The only critical point is $\boxed{(x,y) = (2,1)}$.

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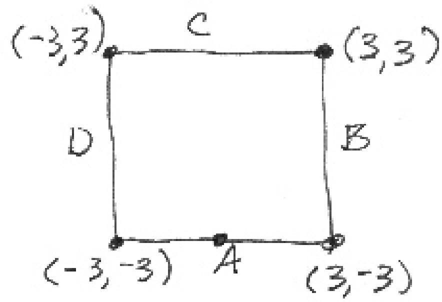
② The global max. and min. must occur either at a critical point or on the boundary. We consider the four sides A, B, C, D, as well as the four corners, in addition to the critical point computed in problem ①.



A: $g_1(x) = f(x,-3) = x^2 + 3x + 9 - 3x = x^2 + 9$

$\frac{dg_1}{dx} = 2x = 0$ at $x = 0$ $\therefore (0,-3)$ is a point of interest.

B: $g_2(y) = f(3,y) = 9 - 3y + y^2 - 9 = y^2 - 3y$

$\frac{dg_2}{dy} = 2y - 3 = 0$ at $y = 3/2$ $\therefore (3, 3/2)$ is a point of interest

C: $g_3(y) = f(x,3) = x^2 - 3x + 9 - 3x = x^2 - 6x + 9$

$\frac{dg_2}{dy} = 2x - 6 = 0$ at $x = 3$ $\therefore (3,3)$ is a point of interest.

D: $g_4(y) = f(-3,y) = 9 + 3y + y^2 + 9 = y^2 + 3y + 18.$

$\frac{dg_4}{dy} = 2y + 3 = 0$ at $y = -3/2$ $\therefore (-3, -3/2)$ is a point of interest.

2 (continued)

Combining these gives the following candidates for global minimizers and global maximizers

	(x,y)	$f(x,y)$	
critical points	$(2,1)$	-3	← global min
edge points	$(0,-3)$	9	
	$(3,3/2)$	$-9/4 = -2.25$	
	$(3,3)$	0	
	$(-3,-3/2)$	$\frac{63}{4} = 15.75$	
corners not included above	$(-3,-3)$	18	
	$(3,-3)$	18	
	$(-3,3)$	36	← global max

3 The minimum and maximum occurs either at the edge of the region (where $x=0$ or $y=0$) or where $g=c$ is tangent to a contour of f. There is only one point of tangency at about $(x,y) \approx (6,6)$. $f = 400$ there.

The intersection of the edges with $g=c$ are at

$(x,y) \approx (0, 13.5)$, where $f < ~~200~~$ 100, and

$(x,y) \approx (10.5, 0)$, where $f < ~~200~~$ 100.

Thus, the maximum is about 400, and it occurs at $(x,y) \approx (6,6)$, while the minimum is some value between ~~1~~0 and ~~100~~ 100, occuring either at $(x,y) \approx (0, 13.5)$ or $(x,y) \approx (10.5, 0)$.