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1. $$\iint_R e^{x+y}\,dA = \int_{x=0}^{1}\int_{y=0}^{1} e^{x+y}\,dy\,dx = \int_{x=0}^{1}\int_{y=0}^{1} e^{x}e^{y}\,dy\,dx$$

$$= \left\{\int_{x=0}^{1} e^{x}\,dx\right\}\left\{\int_{y=0}^{1} e^{y}\,dy\right\} = \left(\int_{x=0}^{1} e^{x}\,dx\right)^2 = \left(e^{x}\Big|_{x=0}^{1}\right)^2 = \boxed{(e-1)^2}$$

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2. The area of the unit disk is $\pi(1)^2 = \pi$. The average value is thus

$$\frac{1}{\pi}\iint_R (x^2+y^2)\,dA = \frac{1}{\pi}\int_{r=0}^{1}\int_{\theta=0}^{2\pi} r^2\,(r\,d\theta\,dr)$$

$$= \frac{1}{\pi}\left\{\int_{\theta=0}^{2\pi} d\theta\right\}\left\{\int_{r=0}^{1} r^3\,dr\right\} = \frac{1}{\pi}(2\pi)\left(\frac{1}{4}\right) = \boxed{\frac{1}{2}}$$

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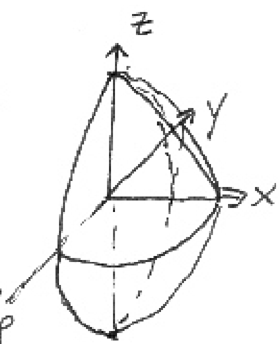
3. The volume is the half of the sphere depicted in the figure. The corresponding spherical coordinates are $0 \le \rho \le 3$, $-\frac{\pi}{2} \le \theta \le \pi/2$, $0 \le \varphi \le \pi$.



The integral becomes:

$$\int_{\rho=0}^{3}\int_{\theta=-\pi/2}^{\pi/2}\int_{\varphi=0}^{\pi} (\rho\sin\varphi\cos\theta)(\rho^2\sin\varphi)\,d\varphi\,d\theta\,d\rho$$

$$= \left\{\int_{\rho=0}^{3} \rho^3\,d\rho\right\}\left\{\int_{\theta=-\pi/2}^{\pi/2} \cos\theta\,d\theta\right\}\left\{\int_{\varphi=0}^{\pi} \sin^2\varphi\,d\varphi\right\}$$

$$= \left\{\frac{81}{4}\right\}\left\{2\right\}\left\{\int_{\varphi=0}^{\pi} \frac{1-\cos 2\varphi}{2}\,d\varphi\right\} = \left\{\frac{81}{4}\right\}\left\{2\right\}\left\{\frac{\pi}{2}\right\}$$

$$= \boxed{\frac{81\pi}{2}}$$

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4 [Volume] $= \int_{x=0}^{2} \int_{y=0}^{x} x\,y\,dy\,dx = \int_{x=0}^{2} x \frac{y^2}{2} \Big|_{y=0}^{x} dx$

$= \int_{x=0}^{2} \frac{x^3}{2} dx = \frac{x^4}{8} \Big|_{x=0}^{2} = \boxed{2}$