

Second Examination
Thursday, March 7, 2013

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 25 points.

1. If

$$f(x, y) = (x + 2y)^5,$$

compute

$$(a) \frac{\partial f}{\partial x}, \quad (b) \frac{\partial f}{\partial y}, \quad (c) \frac{\partial^2 f}{\partial x^2}, \quad (d) \frac{\partial^2 f}{\partial y^2}, \quad \text{and} \quad (e) \frac{\partial^2 f}{\partial x \partial y}.$$

2. Let f be as in Problem 1.

(a) Write down the equation of the tangent plane to f at $(1, 0)$.

(b) Write down the equation of the degree 2 Taylor polynomial to f at $(1, 0)$.

3. Let f be as in Problems 1 and 2. Compute the directional derivative of f at $(1, 0)$ in the direction of $(1, 1)$.

4. Let f be as in problems 1, 2, and 3, and suppose you were at the point $(x, y) = (1, 0)$. State the answers to parts (a), (b), and (c) as unit vectors.

(a) In what direction would you go if you wanted to go in the direction of maximum increase in f ?

(b) In what direction would you go if you wanted to go in the direction of maximum decrease in f ?

(c) In which two directions could you go if you wanted walk along a level curve of f ?

(d) What is the rate of change of f at $(1, 0)$ in the direction of maximum increase in f ?