1. $f(x,y) = (x+2y)^5$

(a) $\frac{\partial f}{\partial x} = \boxed{5(x+2y)^4}$; (b) $\frac{\partial f}{\partial y} = 5(x+2y)^4 \cdot (2) = \boxed{10(x+2y)^4}$

(c) $\frac{\partial^2 f}{\partial x^2} = \boxed{20(x+2y)^3}$; (d) $\frac{\partial^2 f}{\partial y^2} = 40(x+2y)^3 \cdot 2 = \boxed{80(x+2y)^3}$.

(e) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left[10(x+2y)^4\right] = \boxed{40(x+2y)^3}$

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2. $f_x(1,0) = 5$, $f_y(1,0) = 10$, $f_{xx}(1,0) = 20$, $f_{yy}(1,0) = 80$
$f_{xy}(1,0) = 40$, and $f(1,0) = 1$, so

(a) The tangent plane has equation:

$\boxed{z - 1 = 5(x-1) + 10(y-0)}$, or $\boxed{5x + 10y - z = 4}$

(b) $\boxed{z - 1 = 5(x-1) + 10(y) + \frac{1}{2}(20)(x-1)^2 + 40(x-1)y + \frac{1}{2}(80)y^2}$

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3. A unit vector in the required direction is
$\vec{u} \quad \frac{1}{\|(1,1)\|}(1,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, while $\nabla f(1,0) = (5,10)$.

Thus, $f_{\vec{u}}(1,0) = \nabla f(1,0) \circ \vec{u} = (5,10) \circ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \boxed{\frac{15}{\sqrt{2}}} = \boxed{\frac{15}{2}\sqrt{2}}$

4(a) The direction of maximum increase would be

$$\frac{1}{\|\nabla f(1,0)\|} \nabla f(1,0) = \frac{1}{\|(5,10)\|}(5,10) = \frac{1}{\sqrt{125}}(5,10)$$

$$= \boxed{\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)}$$

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4 b The direction of maximum decrease would be

$$-\frac{1}{\|\nabla f(1,0)\|}\nabla f(1,0) = \boxed{\left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)}$$

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4 c The directions are directions perpendicular to ∇f, namely, $\boxed{\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right) \text{ and } \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)}$,

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4 d The maximal rate of change of f is

$$\|\nabla f(1,0)\| = \sqrt{125} = \boxed{5\sqrt{5}}.$$