Math. 302-03 Third exam answers, page 1. Thursday 4/11/2013

① $\nabla f = (1,2)$. $\nabla g = (2x_1, 2x_2)$

$$\left.\begin{array}{l} 1 = 2x_1\lambda \\ 2 = 2x_2\lambda \end{array}\right\} \Rightarrow 4x_1\lambda = 2x_2\lambda \Leftrightarrow 2\lambda(2x_1 - x_2) = 0.$$

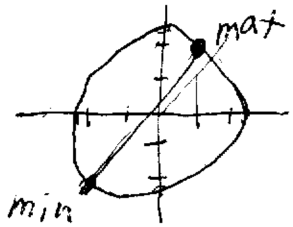
Since $\lambda \neq 0$ (otherwise $1=0$), we have $x_2 = 2x_1$.

Plugging this into the constraint gives:

$$x_1^2 + 4x_1^2 = 5 \Rightarrow x_1^2 = 1 \Rightarrow x_1 = 1, x_2 = 2 \text{ or } x_1 = -1, x_2 = -2.$$

Thus, the maximum and minimum must occur at these two points:

(x_1, x_2)	$x_1 + 2x_2$
$(1,2)$	$5 \leftarrow$ max
$(-1,-2)$	$-5 \leftarrow$ min



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②
$$\int_{\mathcal{V}} x^2+y^2+z^2\,dV = \int_{\rho=0}^{1}\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi} \rho^2(\rho^2\sin\phi)\,d\phi\,d\theta\,d\rho$$

$$= \left\{\int_{\rho=0}^{1}\rho^4\,d\rho\right\}\left\{\int_{\theta=0}^{2\pi}d\theta\right\}\left\{\int_{\phi=0}^{\pi}\sin\phi\,d\phi\right\} \text{ (after simplification)}$$

$$= \left(\frac{1}{5}\right)(2\pi)(2) = \boxed{\frac{4}{5}\pi}$$

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③ The Jacobian is:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 10 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 10 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= (1)(50-48) - 2(40-42) + 3(32-35)$$

$$= (1)(2) - 2(-2) + 3(-3)$$

$$= \boxed{-3}.$$