Math. 302-02
Spring, 2015
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# Fourth Examination

*Tuesday,April 14, 2015*

**Instructions:** This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet as you hand in your exam. Each problem is worth 20 points.

1. Write down parametric equations for the line through the point $(1, 2, 3)$ and in the direction of $(1, -1, 2)$ (that is, in the direction of $\vec{i} - \vec{j} + 2\vec{k}$).

2. A particle travels along the line $x = 1 + t$, $y = 2 - t$, $z = 3 + 2t$, where $t$ is in seconds and $x$, $y$, $z$ are in meters.

   (a) Where and when does the particle hit the plane $x + y + z = 20$?

   (b) How fast is the particle going when it hits the plane?

3. Consider the vector field $\vec{F}(\vec{r}) = x\vec{i}$, where $\vec{r} = x\vec{i} + y\vec{j}$ (that is $\vec{F}$ is defined in the plane).

   (a) Sketch the vector field within the rectangle $x \in [-2, 2]$, $y \in [-2, 2]$.

   (b) Do you expect the line integral over the top half of the unit circle $x^2 + y^2 = 1$, oriented counter-clockwise, to be positive, negative, or 0? Explain.

   (c) Do you expect the line integral over the entire unit circle $x^2 + y^2 = 1$, oriented counter-clockwise, to be positive, negative, or 0? Explain.

4. Consider $\vec{F}(\vec{r}) = x\vec{i} + y\vec{j}$. Use the fact that $\vec{F}$ is a gradient field to compute the line integral of $F$ over $\mathcal{C}$, where

   $$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3,$$

   where

   - $\mathcal{C}_1$ is the portion of the unit circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(0, 1)$.
   - $\mathcal{C}_2$ is the line segment from $(0, 1)$ to $(1, 1)$.
   - $\mathcal{C}_3$ is the portion of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$.

5. Consider the vector field $\vec{F} = y\vec{i}$. Use Green's theorem to compute

   $$\int_{\mathcal{C}} \vec{F}(\vec{r}) \cdots d\vec{r},$$

   where $\mathcal{C}$ is the unit circle $x^2 + y^2 = 1$, oriented counter-clockwise.