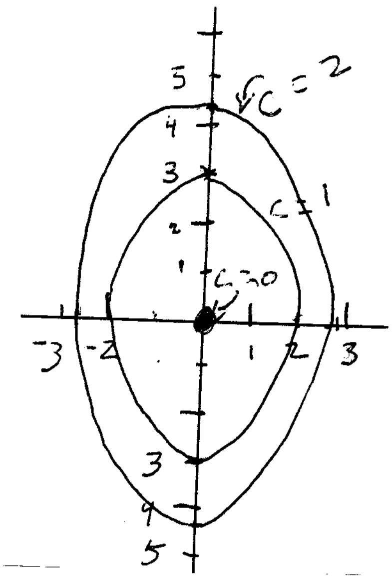
First exam answers, 2/23/2015



(2) We complete the squares:

$$x^2-4x+4+y^2+2y+1+z^2-6z+9=4+1+9$$

$$(x-2)+(y+1)^2+(z-3)^2=14$$

The center is at $(2,-1,3)$ and the radius is $\sqrt{14}$.

(3) We simply form a perpendicular from the ~~components~~: coefficients:

$$\boxed{\vec{N}=(1,-1,2)}.$$

(4) $$\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & -1\\ 0 & 1 & 0\end{vmatrix}=\vec{i}\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}-\vec{j}\begin{vmatrix}1 & -1\\ 0 & 0\end{vmatrix}+\vec{k}\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}$$

$$=\vec{i}(1)-\vec{j}(0)+\vec{k}(1)=\boxed{(1,0,1)}.$$

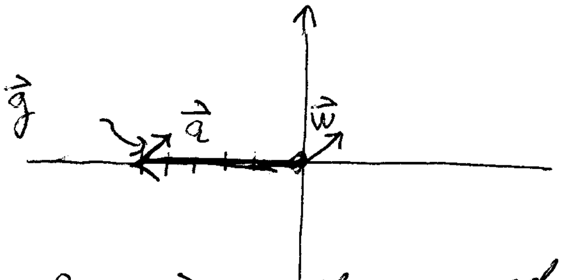
(5) Two displacement vectors in the plane are $\vec{v}=(1,1,0)-(1,0,0)=(0,1,0)$ and $\vec{w}=(1,1,1)-(1,0,0)=(0,1,1)$. A normal to the plane is thus:

$$\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 1 & 0\\ 0 & 1 & 1\end{vmatrix}=\vec{i}\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}-\vec{j}\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix}+\vec{k}\begin{vmatrix}0 & 1\\ 0 & 1\end{vmatrix}=\vec{i}=(1,0,0)$$

Thus, an equation for the plane is:

$$(1,0,0)\circ(x-1,y-0,z-0)=0, \text{ i.e. } \boxed{x=1}.$$

(6)



$\vec{a} = (-500, 0)$ (The air velocity)

$\vec{w} = (100\cos(\pi/4), 100\sin(\pi/4))$ (the wind velocity)

Let $\vec{g}$ be the ground velocity. We have

$$\vec{g} = \vec{a} + \vec{w} = \left(-500 + 100\tfrac{\sqrt{2}}{2},\ 100\tfrac{\sqrt{2}}{2}\right) \approx (-500+71, 71)$$

The ground speed is thus

$$\|\vec{g}\| \approx \sqrt{(-429)^2 + (71)^2} \approx \boxed{435 \text{ kilometers per hour}}$$