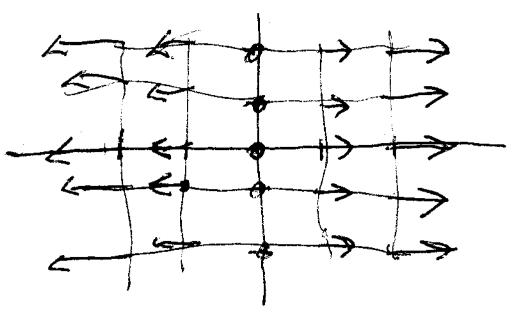
Math. 302-02, Fourth exam answers, 4/14/15, page 1

① $\vec{r}(t) = (1,2,3) + t(1,-1,2)$, that is
$x(t) = 1+t$, $y(t) = 2-t$, $z(t) = 3+2t$ (Note: "in the direction of $(1,-1,2)$" could also mean "towards $(1,-1,2)$")

② ⓐ The particle hits the plane when
$(1+t)+(2-t)+(3+2t) = 20.$
Thus $2t = 14$, i.e. $\boxed{t = 7 \text{ seconds}}$
This corresponds to $x = 1+7 = \boxed{8 \text{ meters}}$,
$y = 2-7 = \boxed{-5 \text{ meters}}$, $z = 3+2(7) = \boxed{17 \text{ meters}}$

ⓑ $\vec{r}\,'(t) = (1,-1,2)$. The speed is
$\|\vec{r}\,'(t)\| = \sqrt{1^2+1^2+2^2} = \boxed{\sqrt{6} \text{ meters per second}}$

③ ⓐ



ⓑ The line integral over the top half of the unit circle is zero, because of symmetry about the y-axis and cancellation.

ⓒ The line ~~interval~~ integral over the entire unit circle is 0 because of ⓑ.

④ We compute a potential function. $\frac{\partial f}{\partial x} = x \Rightarrow f(x,y) = \frac{x^2}{2} + g(y)$
$\frac{\partial f}{\partial y} = g'(y) = y \Rightarrow g(y) = \frac{y^2}{2} + C \Rightarrow f(x,y) = \frac{x^2}{2} + \frac{y^2}{2} + C$

Thus $\displaystyle\int_C \vec{F}(\vec{r})\cdot d\vec{r} = f(2,4) - f(1,0) = \frac{2^2}{2} + \frac{4^2}{2} - \left(\frac{1^2}{2} + \frac{0^2}{2}\right)$

$= 2+8-\frac{1}{2} = \boxed{9\tfrac{1}{2}}$

5 $\displaystyle\int_{C} \vec{F}(\vec{r})\circ d\vec{r} = \iint_{R} \operatorname{curl}(\vec{F})\,dA = \iint_{R} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dA$

$\displaystyle = \int_{r=0}^{1}\int_{\theta=0}^{2\pi} (-1)\,r\,d\theta\,dr = -\left[\int_{r=0}^{1} r\,dr\right]\left[\int_{\theta=0}^{2\pi} d\theta\right]$

$= \boxed{-\pi}$