(1) $\frac{\partial f}{\partial x} = 2x\cos(xy) - x^3 y\sin(x,y)$; $\frac{\partial f}{\partial y} = -x^3 \sin xy$ (for $f(x,y) = x^2\cos(xy)$)

$\frac{\partial f}{\partial x}\Big|_{(1,\pi/2)} = -\pi/2$ $\qquad$ $\frac{\partial f}{\partial y}\Big|_{(1,\pi/2)} = -1$, $f(1,\pi/2) = 0$

Thus, an equation for the tangent plane is:

$$\boxed{z - 0 = \frac{-\pi}{2}(x-1) - 1(y - \pi/2)}, \text{ or}$$

$$\boxed{z = \frac{-\pi}{2}x - y + \pi}$$

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(2) $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$. $\frac{\partial f}{\partial x} = 3x^2(10)(x^3+y^2)^9$

$\frac{\partial f}{\partial y} = 2y(10)(x^3+y^2)^9$.

$\frac{\partial x}{\partial t} = e^{t-1}$; $\frac{\partial y}{\partial t} = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)$

$\frac{\partial x}{\partial t}\Big|_{t=1} = 1$ $\frac{\partial y}{\partial t}\Big|_{t=1} = 0$. $x(1) = 1$; $y(1) = 1$

Thus, $\frac{df}{dt} = 30x^2(x^3+y^2)^9 \cdot 1 + 20y(x^3+y^2)^9 \cdot 0$
$= 30(2^9) = \boxed{15360}$

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(3) For maxima and minima, we look for closed contours. Maxima are where the closed contours increase in value away from the center. Thus, local minima appear to occur at $(-2,-1)$ and at $(2,1)$. A saddle point appears to occur at $(0,0)$. No local maxima are seen.

4 $\nabla f = (2x, 2y)$; $\nabla g = (1,1)$

a $\nabla f = \lambda \nabla g \iff \left.\begin{array}{l} 2x = \lambda \\ 2y = \lambda \end{array}\right\} \Rightarrow x = y.$

Plugging $x = y$ into the constraint gives

$x + x = 1 \Rightarrow \boxed{x = y = 1/2}$ $f(1/2, 1/2) = \frac{1}{2}$

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b