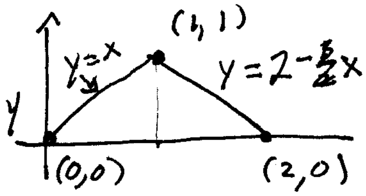
(1)



We will let y correspond to the outer integral, so we only have one double integral to compute.

We have: $0 \le y \le 1$ and $y \le x \le \cancel{4-2}\ 2-y$.

Thus $\iint_{\mathcal{a}} y\, dA = \int_{y=0}^{1}\left[\int_{x=y}^{2-y} y\, dx\right] dy = \int_{y=0}^{1} y\left[(2-y)-y\right] dy$

$= \int_{y=0}^{1} y(2-2y)\,dy = \left[y^2 - \frac{2}{3}y^3\right]\Big|_0^1 = \boxed{\frac{1}{3}}$

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(2)

$\iint_{\mathcal{D}} \sin(x^2+y^2)\, dA = \int_{r=0}^{\sqrt{\pi}} \int_{\theta=0}^{2\pi} \sin(r^2)\, r\, d\theta\, dr$

$= \left[\int_{\theta=0}^{2\pi} d\theta\right]\left[\int_{r=0}^{\sqrt{\pi}} \sin(r^2)\, r\, dr\right] \longleftarrow \boxed{\begin{array}{l} u = r^2 \\ \frac{du}{2} = r\,dr \end{array}}$

$= 2\pi \int_{u=0}^{\pi} \sin(u)\left(\frac{du}{2}\right) = \pi \int_{u=0}^{\pi} \sin u\, du = \pi\left[-\cos u\right]\Big|_{u=0}^{\pi} = \boxed{2\pi}$

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(3) Using cylindrical coordinates, we have:

$\iiint_{\mathcal{V}} e^{x^2+y^2+z}\, dV = \int_{z=0}^{2}\int_{r=0}^{1}\int_{\theta=0}^{2\pi} e^{r^2} e^{z}\, r\, d\theta\, dr\, dz$ $\longleftarrow$ (after taking "constants" out)

$= \left[\int_{\theta=0}^{2\pi} d\theta\right]\left[\int_{r=0}^{1} e^{r^2} r\, dr\right]\left[\int_{z=0}^{2} e^{z}\, dz\right] = 2\pi\left[\frac{1}{2}\int_{u=0}^{1} e^{u}\, du\right]\left[\int_{0}^{2} e^{z}\, dz\right]$

$= \boxed{\pi\left[e^{1}-1\right]\left[e^2-1\right]}$

4. The integral is simply:

$$\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi}\int_{\rho=1}^{4}\frac{1}{\rho^2}(\rho^2\sin\varphi)\,d\rho\,d\varphi\,d\theta$$

$$=\left[\int_{\theta=0}^{2\pi}d\theta\right]\int_{\varphi=0}^{\pi}\int_{\rho=1}^{4}\sin\varphi\,d\rho\,d\varphi=2\pi\int_{\varphi=0}^{\pi}3\sin\varphi\,d\varphi$$

$$=(2\pi)3\int_{\varphi=0}^{\pi}\sin\varphi\,d\varphi=(2\pi)(3)(2)=\boxed{12\pi}\,.$$