Math. 302-02
Summer, 2015
R. B. Kearfott

Final Examination

Thursday, July 30, 2015, 10:15–12:45

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 16 points, and 4 points are free.

1. Compute
$$\int\int\int_{\mathcal{V}} e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, dV,$$
where $\mathcal{V}$ is the volume enclosed by the xy-plane and the top half of the sphere of radius 2 centered at the origin.

2. Compute the place where the line through the points $(1,1,1)$ and $(2,3,4)$ intersects the plane given by $x+y+z=1$.

3. Compute the flux of the vector field
$$\vec{F}(x,y,z) = (x, y, \ln(\sin(e^{x^2-2y+\cos(z)}))$$
through the vertical side of the cylinder centered on the z-axis between $z=-1$ and $z=1$ and with radius 3.

4. Compute the flux of the vector field $\vec{F}(x,y,z) = (x+2xy, -y^2, z)$ out of an oddly-shaped blob in space whose volume has been calculated to be 5.

5. If $\vec{F}(x,y,z) = (yz, xz, xy)$, compute $\int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$, where $\mathcal{C}$ is the path starting at $(0,0,0)$, following a straight line to $(e^9,0,0)$, following the portion of the circle in the xy-plane of radius e^9 centered at 0 from $(e^9,0,0)$ to $(0,e^9,0)$, following the line segment between $(e^9,0,0)$ to $(1,1,1)$, and ending at $(1,1,1)$.

6. Compute $\int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$, where $\mathcal{C}$ is the circle in the plane parallel to the yz-plane given by $y^2+z^2=1$, $x=2$, oriented counterclockwise when viewed from the positive x axis, and $\vec{F}(x,y,z) = (x,x,z)$ in two ways: (a) directly, and (b) using Stokes' theorem.