

Second Examination

Tuesday, July 7, 2015

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 14 points, and 2 points are free.

1. Find $\frac{\partial g}{\partial t}$ and $\frac{\partial g}{\partial u}$ if $g(t, u) = (\sin(t) + e^u)^{10}$.
2. Write down an equation for the tangent plane to $f(x, y) = x^2y$ at $x = 1$, $y = -1$.
3. Compute $D_{\vec{u}}f(1, -1)$, where f is as in Problem 2 and $\vec{u} = (1/\sqrt{5}, 2/\sqrt{5})$.
4. Compute $\frac{df}{dt}\big|_{t=\frac{\pi}{2}}$ if $f(x, y) = (e^x + y)^{10}$, $x = \cos(t)$, and $y = \sin(t)$.
5. Find all critical points of the following function, and state whether or not each critical point you found corresponds to a local maximum or local minimum.

$$f(x, y) = x^2 + xy + y^2.$$

6. The sides of the rectangular secure compartment of an armored vehicle cost \$200 per square meter to construct, the front and back cost \$100 per square meter, and the top and bottom cost \$50 per square meter. What should the dimensions of the armored compartment be to minimize the total cost, if it should hold at least 10 cubic meters?
7. Use Lagrange multipliers to find the maximum and minimum of $f(x, y) = x + y$ subject to $x^2 - y^2 = 1$, if a maximum and minimum exist. If no maximum or minimum exists, show why. Be sure to show all work.