Final Exam
Monday, December 2, 7:30AM

This exam is open book, computer on. Make sure your name is on all pages, and put your name in a text cell on any Mathematica notebooks before you print them. Each problem is worth 20% of the total.

1. Plot a direction field for the equation

$$y' = y^3 + 4y^2 + y - 6.$$

   Label all critical points on the y axis, and indicate for each critical point whether it is stable or unstable. Be sure to use ranges on your plot that include all critical points and that show the vector field around these critical points clearly.

2. Use pencil and paper to derive the solution to the initial value problem

$$\frac{dy}{dt} + \frac{1}{t}y = t, \qquad y(1) = 1.$$

   Check your solution with Mathematica.

3. Find the solution, by hand, to the initial value problem

$$y'' + 25y = \sin(5t), y(0) = 1, y'(0) = 0.$$

   Check your solution with Mathematica.

4. Using hand calculations, find the terms in the Taylor polynomial expansion up to and including degree 4 to the solution to the following initial value problem.

$$y''(t) + ty'(t) = \sin(t), \qquad y(0) = 0, \quad y'(0) = 0.$$

   Check your result with Mathematica.

5. Either by hand or using Mathematica, use Laplace transforms to solve the initial value problem

$$y' + y = \sin(t), \qquad y(0) = 1.$$