1 $y = \sum_{n=0}^{\infty} a_n x^n, \; y' = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n, \; y'' = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n.$

So $(1-x)y'' + xy' - y = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^{n+1}$

$+ \sum_{n=0}^{\infty} (n+1) a_{n+1} x^{n+1} - \sum_{n=0}^{\infty} a_n x^n$

$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=1}^{\infty} (n+1)(n) a_{n+1} x^n + \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} a_n x^n$

$= 2a_2 - a_0 + \sum_{n=1}^{\infty} [(n+2)(n+1) a_{n+2} - (n+1)(n) a_{n+1} + n a_n - a_n] x^n = 0.$

Thus, we have

$2a_2 - a_0 = 0 \Rightarrow a_2 = a_0/2$, and

$(n+2)(n+1) a_{n+2} - (n+1)(n) a_{n+1} + (n-1) a_n = 0,$

whence

$$a_{n+2} = \frac{(n+1)(n) a_{n+1} - (n-1) a_n}{(n+2)(n+1)}, \quad n \geq 1$$

This gives

$\underline{n=1}: \; a_3 = \frac{2a_2}{6} = \frac{a_2}{3} = \frac{a_0}{6}$

$\underline{n=2}: \; a_4 = \frac{6a_3 - a_2}{12} = \frac{6(a_0/6) - a_0/2}{12} = \frac{a_0}{24}.$

$\underline{n=3}: \; a_5 = \frac{12a_4 - 2a_3}{20} = \frac{12\left(\frac{a_0}{24}\right) - \frac{2a_0}{6}}{20} = \frac{\frac{a_0}{2} - \frac{a_0}{3}}{20} = \frac{\frac{a_0}{6}}{20} = \frac{a_0}{120}.$

Thus, the 5-th degree approximation to the solution is:

$$\boxed{y(x) \approx a_0\left\{1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right\} + a_1 x}$$

(2) The singular points occur where $x \sin x = 0$. This gives: $x = 0$ if $x \in [-\pi/2, \pi/2]$. To determine the regularity, we take limits:

$$\lim_{x \to 0}\left(x \frac{Q}{P}\right) = \lim_{x \to 0} \frac{x(3)}{x \sin x} = \lim_{x \to 0} \frac{3}{\sin x}$$ ~~is~~ is not finite.

Therefore, this singular point is irregular.

(3) This is an Euler equation, with indicial equation: $\cancel{\alpha(\alpha-1)+5\alpha+1=0}$, i.e. $\alpha^2 - 5\alpha + 4 = 0$

$\alpha(\alpha-1) - 4\alpha + 4 = 0 \iff (\alpha-4)(\alpha-1) = 0.$

Thus, the general solution is:

$$\boxed{y(x) = c_1 x^4 + c_2 x}.$$