① For a first-order linear equation, ~~the~~ an integrating factor is: $u(t) = e^{\int p(t)\,dt}$. In this case, $p(t) \equiv -1$, so $u(t) = e^{-t}$.

$(e^{-t}y)' = 1 \Rightarrow e^{-t}y(t) = t + C \Rightarrow$

$y(t) = e^{t}(t+C) = te^{t} + Ce^{t}$.

$y(0) = 0 = 0 + C \Rightarrow C = 0 \Rightarrow \boxed{y(t) = te^{t}}$

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② $\alpha - 1 = 0 \Rightarrow \alpha = 1 \Rightarrow y_h(t) = Ce^{t}$.

Since the right-hand side is also a solution to the homogeneous equation, we assume a solution of the form ~~$y(t) = Ate^{t} + B$ Plugging this in~~, $y(t) = Ae^{t} + Bte^{t}$

~~$y'(t) = C(e^{t} + te^{t})$ so~~ $y'(t) = Ae^{t} + Be^{t} + Bte^{t}$, so

~~$y' - y = Ce^{t}$~~

$y' - y = (A+B)e^{t} + Bte^{t} - Ae^{t} - Bte^{t} = Be^{t} = e^{t}$

$\Rightarrow B = 1$, A is anything.

Thus, the general solution is:

$y_g(t) = te^{t} + Ce^{t}$.

As in problem ①, we get $C = 0$, and $\boxed{y(t) = te^{t}}$

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3 $\mathcal{L}(y' - y) = \mathcal{L}(e^t)$

$sY - y(0) - Y = \dfrac{1}{s-1}$, where we used 2 and 18 of the table.

$(s-1)Y = \dfrac{1}{s-1}$, whence $Y = \dfrac{1}{(s-1)^2}$

Now, using 11 of the table, we obtain ($n=1, a=1$)

$$\boxed{y(t) = te^t}$$