Math. 350-03, First exam answers, Monday 2007/09/10

1 (a) Linear - y and y'' occur linearly.

(b) Nonlinear since e^y is a nonlinear term in the dependent variable.

(c) Linear: The dependent variable and all of its derivatives occur linearly.

(d) Nonlinear, since y', the derivative of the dependent variable, occurs nonlinearly.

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2 (a) Second order

(b) First order

(c) Fourth order

(d) First order.

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3 $u = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$

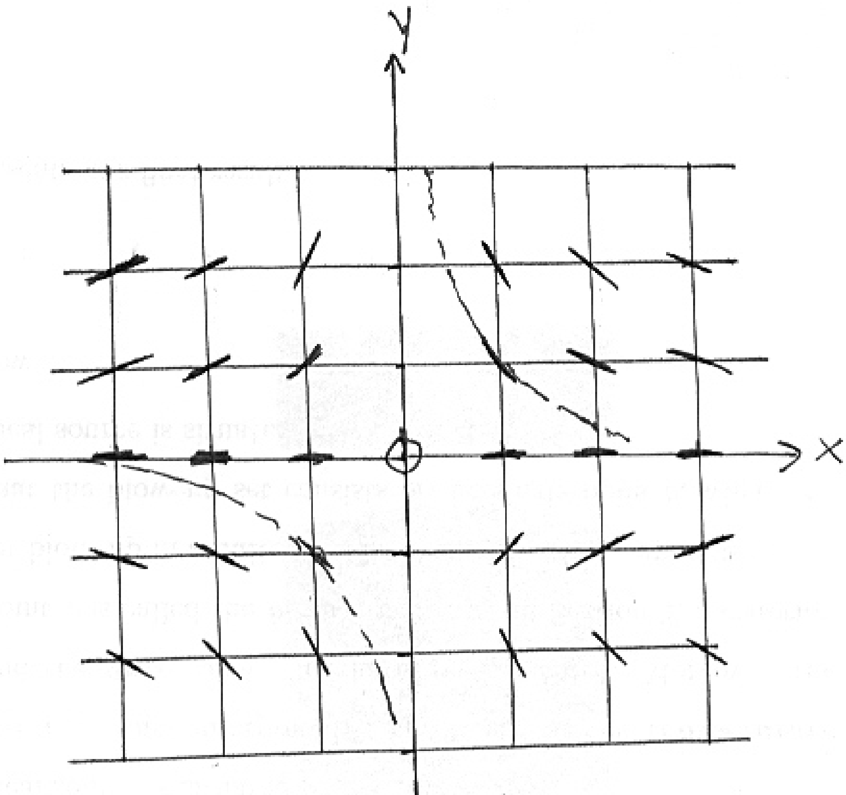
$t(y' + \frac{1}{t} y) = t, \quad (ty)' = t \Rightarrow$

$ty = \frac{t^2}{2} + C \Rightarrow y(t) = \frac{t}{2} + \frac{C}{t}.$

$y(1) = \frac{1}{2} = \frac{1}{2} + \frac{C}{1} \Rightarrow C = 0.$

$$\boxed{y(t) = \frac{t}{2}}$$

4



$$\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\Rightarrow \ln(y) = -\ln x + c \Rightarrow y = \frac{K}{x}.$$

The solutions are hyperbolas skewed 45° with respect to the coordinate axes. This can be seen from the direction field, where the flow turns asymptotically parallel to the coordinate axes.