

**Final Exam**

*Thursday, December 11, 2:00PM to 4:30PM*

This exam is closed book. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each part of each problem is worth 25 points. You may keep this sheet after turning in your exam.

1. Solve the following initial value problems.

(a)  $\frac{dy}{dx} + \frac{1}{x}y = x, \quad y(1) = 1.$

(b)  $y'' + 4y = \cos(2t), \quad y(0) = 0, \quad y'(0) = 0.$

2. Write down the terms up to and including degree 5 of a series solution to the following initial value problem.

$$y' + (x - 1)y = (x - 1), \quad y(1) = 0.$$

3. Solve

$$y' + y = e^{-t}, \quad y(0) = 0$$

using Laplace transforms. You may use Table 1 (attached).

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. $e^{at}$	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s >  a $	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s >  a $	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	$e^{-cs}$	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28

Table 1: From W. E. Boyce and R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, eighth edition, Wiley, 2006.