Second Exam
Monday, September 29, 2008

This exam is closed book. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 25 points. You may keep this sheet after turning in your exam.

1. Solve the initial value problem

$$\frac{dy}{dx} = x^2 y^2, \quad y(1) = 1,$$

   and determine the interval in which the solution lies.

2. The population of mosquitoes in a certain area increases at a rate proportional to the current population, and in the absence of other factors, the population doubles each week. There are 100,000 mosquitoes in the area initially, and predators (birds, bats, etc.) eat 15,000 mosquitoes per day. Determine the population of mosquitoes in the area at any time.

3. A cylindrical water tank of constant cross section 250 square meters has water pumped into it at a rate of 1 square meter per minute, while water leaks out of the bottom of the tank through a hole of area 0.01 square meter. It can be derived from physical principles that the depth of water h in the tank at any time t minutes satisfies

$$\frac{dh}{dt} = (1 - 0.0075\sqrt{20h})/250.$$

   Determine the equilibrium depth of the water, and show that it is asymptotically stable.

4. In Problem 3, assume that at time $t = 0$ minutes, there is $h = 10$ meters of water in the tank.

   (a) Do 5 steps of Euler's method, with step size 10, to determine an approximate level of water in the tank after 50 minutes.

   (b) Do you think the water is near its equilibrium level after 50 minutes? How long do you think it would take for the level to closely approximate its equilibrium value?