Fifth Exam

Wednesday, November 26, 2008

This exam is closed book. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. The problem will be graded both on correctness and on how logical your presentation is. You may keep this sheet after turning in your exam.

Solve the following differential equation using Laplace transforms. You may refer to Table 1.

$$y'' + 3y' + 2y = u_2(t)e^{t-2}, \quad y(0) = 0, \quad y'(0) = 1.$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\dfrac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\dfrac{1}{s - a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n = $ positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\dfrac{a}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\dfrac{s}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\dfrac{a}{s^2 - a^2}, \quad s > |a|$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\dfrac{s}{s^2 - a^2}, \quad s > |a|$	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\dfrac{b}{(s - a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\dfrac{s - a}{(s - a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n = $ positive integer	$\dfrac{n!}{(s - a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t) f(t - c)$	$e^{-cs} F(s)$	Sec. 6.3
14. $e^{ct} f(t)$	$F(s - c)$	Sec. 6.3
15. $f(ct)$	$\dfrac{1}{c} F\left(\dfrac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\displaystyle\int_0^t f(t - \tau) g(\tau)\, d\tau$	$F(s) G(s)$	Sec. 6.6
17. $\delta(t - c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28

Table 1: From W. E. Boyce and R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, eighth edition, Wiley, 2006.