$$\int \frac{dy}{y^2} = \int x^2\,dx \Rightarrow -\frac{1}{y} = \frac{x^3}{3} + \tilde{c} \Rightarrow y = \frac{1}{c - x^3/3} = \frac{3}{c - x^3}.$$

$$y(1) = 1 = \frac{3}{c-1} \Rightarrow c - 1 = 3 \Rightarrow c = 4 \Rightarrow y = \frac{3}{4 - x^3}.$$

The solution is defined for $x < \sqrt[3]{4}$.

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(2) The intrinsic rate of growth is $P(t) = Ce^{\alpha t}$, where $\frac{dP}{dt} = \alpha P$. Assuming t is given in weeks, we have $P(1) = Ce^{\alpha} = 2P(0) = C \Rightarrow e^{\alpha} = 2 \Rightarrow \alpha = \ln(2)$.

The differential equation is thus given by:

$$\frac{dP}{dt} = \ln(2)P - 7(15000) = \ln(2)P - 105000, \quad P(0) = 100{,}000.$$

Solving the initial value problem:

$$\int \frac{dP}{\ln(2)P - 105{,}000} = \frac{1}{\ln(2)} \int \frac{dP}{P - 105000/\ln(2)} = \int dt$$

$$\Rightarrow \frac{1}{\ln(2)} \ln\left|P - 105000/\ln(2)\right| = t + c$$

$$\Rightarrow P - 105{,}000/\ln(2) = Ce^{t(\ln(2))} = C2^t$$

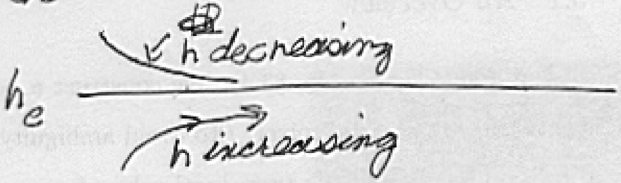
$$P(0) = 100{,}000 \Rightarrow C = 100{,}000 - 105000/\ln(2) \approx -51482$$

Thus, $\boxed{P(t) = \frac{105000}{\ln(2)} + \left(100{,}000 - 105000/\ln(2)\right)2^t}$

(3) The equilibrium depth is where $1-0.0075\sqrt{20h}=0$,

that is, $h_e = \frac{1}{20}\left(\frac{1}{.0075}\right)^2 = \cancel{13.\bar{3}}\ 888.\bar{8}$ meters.

It is asymptotically stable since $\frac{dh}{dt} < 0$ when $h > h_e$ and $\frac{dh}{dt} > 0$ when $h < h_e$



(4) We will make a table of values.

t	$\frac{dh}{dt}$	h
0	$\approx .0035757$	10
10	$\approx .003575$	≈ 10.035753
20	$\approx .003574$	≈ 10.0715
30	$\approx .003573$	≈ 10.10
40	$\approx .003572$	≈ 10.14
50	$\approx .003572$	≈ 10.18

The value is increasing towards the equilibrium of $13.\bar{3}$ meters, and the derivative is decreasing towards 0. However, it is not happening on the scale of 10 minutes, but more likely on the scale of a day or so. Using a stepsize of 100 minutes, rather than 10 minutes, may be more revealing.