① The characteristic equation is $\alpha^2+3\alpha+2=(\alpha+2)(\alpha+1)=0$.
The general solution to the homogeneous system is thus:

$$y_h(t)=C_1e^{-2t}+C_2e^{-t}.$$

A particular solution to the forced system can have the form:

$y_p(t)=A\cos t+B\sin t$. Plugging into the equation, we have:

$$y_p'=-A\sin t+B\cos t$$
$$y_p''=-A\cos t-B\sin t$$

$$\begin{aligned} y_p''+3y_p'+2y_p &= -A\cos t-B\sin t \\ &\quad +3B\cos t-3A\sin t \\ &\quad +2A\cos t+2B\sin t \\ &= (A+3B)\cos t+(-3A+B)\sin t=(0)\cos t+(1)\sin t. \end{aligned}$$

Thus, $\left.\begin{aligned} A+3B&=0 \\ -3A+B&=1 \end{aligned}\right\} \Rightarrow A=-3B \Rightarrow -3(-3B)+B=1 \Rightarrow \boxed{B=\frac{1}{10}}$

$\Rightarrow \boxed{A=-\frac{3}{10}}$.

Thus, the general solution is

$$y_g=C_1e^{-2t}+C_2e^{-t}-\frac{3}{10}\cos t+\frac{1}{10}\sin t$$
$$y_g'=-2C_1e^{-2t}-C_2e^{-t}+\frac{3}{10}\sin t+\frac{1}{10}\cos t$$

$$\left.\begin{aligned} y_g(0)&=C_1+C_2-\frac{3}{10}=0 \\ y_g'(0)&=-2C_1-C_2+\frac{1}{10}=0 \end{aligned}\right. \Rightarrow \begin{aligned} C_1+C_2&=3/10 \\ +[-2C_1-C_2&=-1/10] \\ \hline -C_1&=2/10 \Rightarrow \boxed{C_1=-1/5}. \end{aligned}$$

$$-\frac{1}{5}+C_2=3/10 \Rightarrow C_2=\frac{5}{10}=\boxed{1/2=C_2.}$$

The ~~general~~ solution is thus:

$$\boxed{y(t)=-\frac{1}{5}e^{-2t}+\frac{1}{2}e^{-t}-\frac{3}{10}\cos t+1/10\sin(t)}$$

2. Since e^{-2t} is a solution to the corresponding homogeneous equation, we assume a solution of the form

$$y_p(t) = Ate^{-2t}, \text{ so } y_p' = A(e^{-2t} - 2te^{-2t}),$$

and $y_p'' = A(-2e^{-2t} - 2e^{-2t} + 4te^{-2t})$.

Plugging into the equation gives:

$$-2Ae^{-2t} - 2Ae^{-2t} + 4Ate^{-2t} + 3Ae^{-2t} - 6Ate^{-2t} + 2Ate^{-2t}$$

$$= A[(-2-2+3)e^{-2t} + (4-6+2)te^{-2t}] = -Ae^{-2t} = 1e^{-2t}$$

$$\Rightarrow \boxed{A = -1}.$$

The general solution is thus:

$$y_g(t) = c_1e^{-2t} + c_2e^{-t} - te^{-2t}$$

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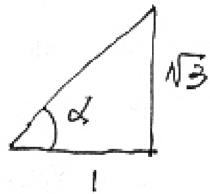
3. $\frac{\sqrt{3}}{2}\cos(10^6x) + \frac{1}{2}\sin(10^6x) = C\sin(\alpha)\cos(10^6x) + C\cos\alpha\sin(10^6x)$.

Whence $C\sin\alpha = \frac{\sqrt{3}}{2}$, $C\cos(\alpha) = \frac{1}{2}$,

whence $C^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$. Choose $C = 1$.



Then $\tan\alpha = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$, whence $\alpha = \pi/3$.

$$\therefore \frac{\sqrt{3}}{2}\cos(10^6x) + \frac{1}{2}\sin(10^6x) = \boxed{\sin\left(10^6x + \frac{\pi}{3}\right)}$$