① $y(x) = \sum_{n=0}^{\infty} [a_n](x-1)^n$. $y'(x) = \sum_{n=1}^{\infty} n a_n (x-1)^{n-1}$

$(x-1)y'(x) = \sum_{n=1}^{\infty} [n a_n](x-1)^n$. $y''(x) = \sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2}$

$$= \sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2}](x-1)^n$$

Thus,

$$y'' + (x-1)y' + y = \underbrace{a_0 + 2a_2 + \sum_{n=1}^{\infty} [a_n + n a_n + (n+2)(n+1)a_{n+2}](x-1)^n}_{=0}.$$

Therefore

$a_0 + 2a_2 = 0 \Rightarrow a_2 = \frac{-1}{2} a_0$

and $a_n + n a_n + (n+2)(n+1)a_{n+2} = (n+1)a_n + (n+2)(n+1)a_{n+2} = 0$, $n \geq 1$.

This gives $a_{n+2} = \frac{-a_n}{n+2}$, $n \geq 1$

From the initial values, we have $a_0 = 1$ and $a_1 = 2$.

Thus, $a_2 = \frac{-1}{2}(1) = \frac{-1}{2}$ and $a_3 = a_3 = \frac{-a_1}{3} = \frac{-2}{3}$

whence the first four terms are

$$\boxed{y(x) \approx 1 + 2(x-1) - \frac{1}{2}(x-1)^2 - \frac{2}{3}(x-1)^3}$$

2 The characteristic equation is:

$$\alpha^3+3\alpha^2+2\alpha=\alpha(\alpha+1)(\alpha+2)=0,$$

whence the general solution is

$$y(t)=c_1+c_2e^{-t}+c_3e^{-2t}. \quad y(0)=1=c_1+c_2+c_3$$

$$y'(t)=-c_2e^{-t}-2c_3e^{-2t}. \quad y'(0)=0=-c_2-2c_3$$

$$y''(t)=c_2e^{-t}+4c_3e^{-2t}. \quad y''(0)=0=c_2+4c_3$$

To find c_1, c_2, and c_3, we add the last two equations to get $2c_3=0 \Rightarrow c_3=0 \Rightarrow c_2=0$.

Plugging into the first equation then gives $c_1=1$, so

$$y(t)\equiv 1.$$