Math. 350-02
Fall, 2012
R. B. Kearfott

Final Exam

2:00PM– 4:30PM
Tuesday, December 4, 2012

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 20 points. You may keep this exam sheet.

1. Consider $y' = -\frac{1}{3}y + 1$.
   (a) Find the general solution to this differential equation.
   (b) Is there an equilibrium value associated with this equation? If so, what is it?
   (c) If there is an equilibrium value, is it stable, unstable, or neither? If so, why?
   (d) Solve the initial value problem corresponding to this differential equation and the initial value $y(0) = 1$.
   (e) If there is an equilibrium value, does the solution to your initial value problem approach this equilibrium value as $t \to \infty$?

2. Find the solution to the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(0) = 1, \; y'(0) = 0.$$

3. Write down the terms up to and including degree 5 to the series solution of

$$x^2y'' + y' + y = 1, \quad y(0) = 0, \; y'(0) = 1.$$

4. (a simplified model of an actual dating technique) Uranium 238 decays to lead 206, with a half-life of approximately 4.5×10^9 years. By measuring the ratios of uranium to lead in certain minerals, geologists can determine the original amount of uranium and thus find out the age of the stratum where the mineral was found. If a particular stratum is found to contain 97.72% (i.e. a fraction of 0.9772) of the original uranium, approximately how old is the deposit?

5. (Refer to Table 1 to do this problem.) Use Laplace transforms to find the solution to

$$y'' + y = f(t), \quad y(0) = 1, \; y'(0) = 0,$$

where

$$f(t) = \begin{cases} 0 & \text{for } t < \pi, \\ \sin(2(t-\pi)) & \text{for } t \geq \pi. \end{cases}$$

Table 1: Table of Laplace Transforms

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\dfrac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\dfrac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n =$ positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n =$ positive integer	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\displaystyle\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^nf(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28