First Exam
Tuesday, September 18, 2012

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner.

1. (25 points) Classify each of the following differential equations as linear or non-linear. In each case, say why.

$$\text{(a) } \frac{d^2y}{dt^2} + e^t y^2 = t. \qquad\qquad \text{(b) } y' + e^t e^y = 0.$$

$$\text{(c) } y''' + 3y'' + y = e^{-t}. \qquad \text{(d) } y'\sin(t) + \sin(y) = e^{-t}.$$

2. (15 points) State the order of each of the equations in Problem 1.

3. (30 points) Consider $y' = 2y - 1$.

   (a) Find the general solution to this differential equation.

   (b) Is there an equilibrium value associated with this equation? If so, what is it?

   (c) If there is an equilibrium value, is it stable, unstable, or neither? If so, why?

   (d) Solve the initial value problem corresponding to this differential equation and the initial value $y(0) = 1$.

   (e) If there is an equilibrium value, does the solution to your initial value problem approach this equilibrium value as $t \to \infty$?

4. (30 points) An accident has released a large amount of radioactive iodine-131 into a building of a medical facility. The amount of radiation, proportional to the amount of iodine-131 present, is initially 1000 times the recommended maximum safe level, while the half-life of iodine-131 is 8 days. How many days will it be until it is safe to occupy the building again?