

- ① (a) Nonlinear, since it contains a y^2 term.
 (b) Nonlinear, since it contains an e^y term.
 (c) Linear.
 (d) Nonlinear, since it contains a $\sin(y)$ term.

- ② (a) second order
 (b) first order
 (c) third order
 (d) first order

③ (a) $y' - 2y = -1$.

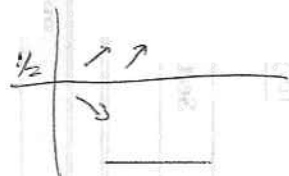
$$(e^{-2t} y)' = -e^{-2t}$$

$$e^{-2t} y = \frac{1}{2} e^{-2t} + C$$

$$y(t) = \frac{1}{2} + C e^{2t}$$

(b) $y' = 2y - 1 = 0$ at $y = \frac{1}{2}$, the equilibrium value.

(c) It is unstable since $y' > 0$ for $y > \frac{1}{2}$ and $y' < 0$ for $y < \frac{1}{2}$.



(d) $y(0) = \frac{1}{2} + C = 1 \Rightarrow C = \frac{1}{2}$

$$\therefore y(t) = \frac{1}{2} + \frac{1}{2} e^{2t}$$

(e) No, the solution diverges from this equilibrium value.

(4) The amount A obeys the D.E.

$A' = -rA \Rightarrow A(t) = A_0 e^{-rt}$, where A_0 is the initial amount.
Since the half-life is 8 days, if t is given in days,
we have $\frac{1}{2} A_0 = A_0 e^{-8r}$, that is, $e^{-8r} = \frac{1}{2}$.

This gives $-8r = \ln(\frac{1}{2}) = -\ln(2) \Rightarrow r = \frac{\ln(2)}{8}$

$$\Rightarrow A(t) = A_0 e^{-\frac{\ln(2)}{8}t} = A_0 (e^{-\ln(2)})^{t/8} = A_0 \left(\frac{1}{2}\right)^{t/8}$$

If T is the minimum time until it is safe
to occupy the building, we have $A_0 \left(\frac{1}{2}\right)^{T/8} = \frac{A_0}{1000}$,

that is, $\left(\frac{1}{2}\right)^{T/8} = \frac{1}{1000}$. Taking logarithms of
both sides gives:

$$\frac{T}{8} \ln\left(\frac{1}{2}\right) = \ln\left(\frac{1}{1000}\right) \Rightarrow T = \frac{8 \ln\left(\frac{1}{1000}\right)}{\ln\left(\frac{1}{2}\right)} = \boxed{\frac{8 \ln(1000)}{\ln(2)}}$$

That is, $T \approx 79.72$ days.

It would be safe to wait 80 days.
