1 (a) Char. eq.: $\alpha^2+25=0 \Rightarrow \alpha=\pm 5i$.

The solution is thus of the form

$y(t) = A\cos(5t) + B\sin(5t)$.

$y(0) = A = 1$.

$y'(t) = -5A\sin(5t) + 5B\cos(5t)$

$y'(0) = 5B = 0 \Rightarrow B = 0$.

$\boxed{\therefore y(t) = \cos(5t)}$

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1 (b) as in (a), $A=0$, $5B=1$, so

$\boxed{y(t) = \frac{1}{5}\sin(5t)}$

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1 (c) We assume a particular solution of the form $y_p(t) = C\cos(t) + D\sin(t)$.

$y_p''(t) = -C\cos(t) - D\sin(t)$

$+25\, y_p(t) \quad 25C\cos(t) + 25D\sin(t)$

$= (24C)\cos(t) + (24D)\sin(t) = \sin(t)$

This gives $24C=0$, $24D=1 \Rightarrow D=\frac{1}{24}$, so the general solution is

$y_g(t) = A\cos(5t) + B\sin(5t) + \frac{1}{24}\sin(t)$.

$y_g(0) = A = 0$

$y_g'(t) = -5A\sin(5t) + 5B\cos(5t) + \frac{1}{24}\cos(t)$

$= 5B + \frac{1}{24} = 0 \Rightarrow B = \cancel{\frac{25}{24}}, -\frac{1}{120}$

Thus, the solution to the initial value problem is:

$\boxed{y(t) = -\frac{1}{120}\sin(5t) + \frac{1}{24}\sin(t)}$

1d Since sin(5t) is a solution to the corresponding homogeneous equation, we must multiply the assumed particular solution by t:

$$y_p(t) = Ct\cos 5t + Dt\sin(5t)$$

$$y_p'(t) = C\cos(5t) + D\sin(5t) - 5Ct\sin(5t) + 5Dt\cos(5t)$$

$$y_p''(t) = -5C\sin(5t) + 5D\cos(5t) - 5C\sin(5t) + 5D\cos(5t)$$
$$- 25Ct\sin(5t) - 25Dt\cos(5t)$$
$$= -10C\sin(5t) + 10D\cos(5t) - 25Ct\overset{\cos}{\cancel{\sin}}(5t) - 25Dt\overset{\sin}{\cancel{\cos}}(5t)$$

$+ 25y_p$: $\qquad 25Ct\cos(5t) + 25Dt\sin(5t)$

$$= -10C\sin(5t) + 10D\cos(5t) = \cancel{10D = 0 \Rightarrow D = 0}\ \sin(5t)$$

$\cancel{y_p'(t)} \Rightarrow -10C = 1,\ C = -\frac{1}{10},\ 10D = 0 \Rightarrow D = 0.$

Thus, $y_g(t) = A\cos(5t) + B\sin(5t) - \frac{t}{10}\cos(5t)$

$$y_g(0) = A - \frac{0}{10} = 0 \Rightarrow A = 0$$

$$y_g'(t) = -5A\sin(5t) + 5B\cos(5t) - \frac{1}{10}\cos(5t) + \frac{5t}{10}\sin(5t)$$

$$y_g'(0) = 5B - \frac{1}{10} = 0 \Rightarrow B = \frac{1}{50}.$$

$$\boxed{y_g(t) = \frac{1}{50}\sin(5t) - \frac{t}{10}\cos(5t)}.$$

(1e) The characteristic equation is:

$$\alpha^2+4\alpha+4=0,\ \text{i.e. } \alpha+2=0,\ \alpha=-t$$

The general solution is thus:

$$y(t)=Ae^{2t}+Bte^{2t}.$$

$$y(0)=1=A$$

$$y'(t)=-2Ae^{-2t}+Be^{2t}-2Bte^{-2t}$$

$$y'(0)=-2A+B=0 \Rightarrow \cancel{B=A=1},\ B=2A=2$$

$$\boxed{\therefore y(t)=e^{-t}+2te^{-t}}$$

---

(2) As in Problem 1, the general solution is

$$Q(t)=A\cos(5t)+B\sin(5t)$$

$$Q(0)=A=1$$

$$Q'(t)=-5A\sin(5t)+5B\cos(5t)$$

$$=5B=5 \Rightarrow B=1,$$

so

$$Q(t)=\cos(5t)+\sin(5t)$$

$$=A\cos(5t-\delta) \quad \text{(a different "A" from above)}$$

$$=A\cos(5t)\cos(\delta)+A\sin(5t)\sin(\delta)$$

$$A\cos(\delta)=1,\ A\sin(\delta)=1$$

$$\Rightarrow \delta=\pi/4,\ A=\sqrt{1+1}=\sqrt{2}.$$

the amplitude is $\sqrt{2}$, the angular velocity is 5, the phase shift is $\pi/4$, and the period is $\frac{2\pi}{5}$.

$$\left(Q(t)=\sqrt{2}\cos(5t-\pi/4)\right)$$