Math. 350-01
Fall, 2013
R. B. Kearfott

Final Exam

Thursday, December 12, 2013, 2:00PM to 4:30PM

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. You may keep this question sheet when you leave.

1. An object weighing 288 pounds is supported by a spring. When the object is placed on the spring, the spring is pressed down 6 inches. A large bump to the system moves the object down. When the object is down 1 ft. from its resting position, it is observed to have an upward velocity of 8 ft./sec. at which time it comes to rest (at time $t = 0$), before starting to oscillate.

   (a) Write down a function giving the location $y(t)$ of the object at time $t \geq 0$.

   (b) Calculate the natural frequency ω, period T, amplitude R, and phase shift δ of the mass' oscillations, where the oscillation is of the form

   $$y(t) = R\cos(\omega t - \delta).$$

   In your calculations, if the acceleration of gravity is needed, use 32 feet per second per second to approximate the acceleration due to gravity.

2. Consider the initial value problem

   $$y'' + x^2 y' + y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

   (a) Write down a recursion relation for the coefficients a_n of the power series solution

   $$y(x) = \sum_{n=0}^{\infty} a_n x^n.$$

   (b) Write down the terms up to and including the x^4 term for the solution to the initial value problem.

3. Use the supplied table of Laplace transforms to solve

   $$y'' + 5y' + 4y = 2\delta(t-5), \quad y(0) = 0, \quad y'(0) = 0.$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. t^n, $n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28