Math. 350-01
Fall, 2013
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First Exam
Friday, September 20, 2013

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 25 points.

1. Write down a differential equation of the form

$$\frac{dy}{dt} = ay + b$$

all of whose solutions approach $y = 4$ as $t \to \infty$.

2. Classify each of the following differential equations as linear or nonlinear.

(a) $\dfrac{d^2y}{dt^2} + e^t \dfrac{dy}{dt} + 2y = t.$ (b) $y' + e^t e^y = 0.$

(c) $y''' + 3y'' + y = e^{-t}.$ (d) $\dfrac{dy}{dx} = x^2 y^2.$

3. Find the solution to the initial value problem

$$\frac{dy}{dt} = t - y + 1, \quad y(0) = 0.$$

4. A swimming pool containing 500,000 gallons of water has a chlorine concentration of 5 units per gallon. Normally, the maximum reasonable chlorine concentration is 3 units per gallon. A fire hose is placed at one end of the pool, and delivers 500 gallons per minute of city water that contains 1 unit of chlorine per gallon, while thoroughly mixed water exits the pool at the same rate through a drain at the other end.

   (a) Write down an initial value problem for the total amount $Q(t)$ of chlorine (in total units) t minutes after the fire hose starts flowing.

   (b) Solve that initial value problem.

   (c) Will the concentration of chlorine in the pool be at an acceptably low level after 24 hours? Show why.