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(1) $\frac{dy}{dt} = -y + 4$ is one such equation.

(2) (a) linear; (b) nonlinear; (c) linear; (d) nonlinear

(3) $\frac{dy}{dt} + y = t + 1$. An integrating factor is $u(t) = e^{\int 1\,dt} = e^t$

$(e^t y)' = te^t + e^t$, so $e^t y = \int te^t\,dt + \int e^t\,dt$,

$e^t y = te^t - e^t + e^t + C = te^t + C$, so

$y(t) = t + Ce^{-t}$. $y(0) = 0$ gives $0 = C$, so $\boxed{y(t) = t}$

4(a) $\frac{dQ}{dt} = 1(500) - \frac{Q}{500{,}000}(500) = 500 - \frac{Q}{1000}$, $Q(0) = 5(500{,}000)$

(b) $\frac{dQ}{dt} + \frac{Q}{1000} = 500$, $u = e^{t/1000}$, so $(e^{t/1000} Q)' = 500e^{t/1000}$

$\Rightarrow Q(t) = e^{-t/1000}\left(500(1000)e^{t/1000} + C\right)$

$= 500{,}000 + Ce^{-t/1000}$.

$Q(0) = 5(500{,}000) = 500{,}000 + C \Rightarrow C = 4(500{,}000)$

(c) The concentration at time t is $Q/500{,}000 = 1 + 4e^{-t/1000}$

24 hours is 1440 minutes, so the concentration after 24 hours is

$$1 + 4e^{-1.440} \approx 1.9477 < 3.$$

Thus, the pool water will be acceptable after 24 hours.