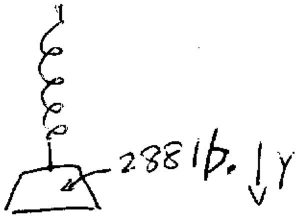

$my'' + ky = 0, \; y(0) = 1, \; y'(0) = -8.$

$m = F/g = 288/32 = 9 \text{ slugs}$

$k = \text{Force/displacement} = 288/\tfrac{1}{2} = 576 \text{ lb/ft}.$

$9y'' + 576y = 0 \Leftrightarrow y'' + 64y = 0$

$r^2 + 64 = 0 \Rightarrow r = \pm 8i$, so

$y(t) = C_1 \cos(8t) + C_2 \sin(8t)$

$y(0) = \boxed{C_1 = 1}$; $y'(0) = 8C_2 = -8 \Rightarrow \boxed{C_2 = -1}.$

$\boxed{\text{(a)} \quad \therefore y(t) = \cos(8t) - \sin(8t)}$

$R\cos(\omega t - \delta) = R\cos\delta\cos(\omega t) + R\sin(\delta)\sin(\omega t)$

$R\cos(\delta) = 1, \; R\sin(\delta) = -1.$

$\Rightarrow \boxed{R = \sqrt{2}}.$ $\tan(\delta) = -1$, and δ is a ~~the~~ second-quadrant angle, so $\boxed{\delta = 3\pi/4}.$

(b) $\boxed{\therefore y(t) = \sqrt{2}\cos(8t - 3\pi/4)}$

(a) $y(x) = \sum_{n=0}^{\infty} a_n x^n$, $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$, $y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$x^2 y' = \sum_{n=1}^{\infty} n a_n x^{n+1} = \sum_{n=2}^{\infty} (n-1) a_{n-1} x^n.$$

Thus, $y'' + x^2 y' + y = 2a_2 + 6a_3 x + a_0 + a_1 x$

$$+ \sum_{n=2}^{\infty} \left[(n+2)(n+1) a_{n+2} + (n-1) a_{n-1} + a_n\right] x^n = 0$$

This gives:

$a_0 + 2a_2 = 0 \Rightarrow \boxed{a_2 = -a_0/2}$

$a_1 + 6a_3 = 0 \Rightarrow \boxed{a_3 = -a_1/6}$

$$\boxed{a_{n+2} = \frac{-a_n - (n-1) a_{n-1}}{(n+2)(n+1)} \quad n \geq 2}$$

(b) $a_0 = 1$, $a_1 = 0$, $a_2 = -1/2$, $a_3 = 0$

$$a_4 = \frac{-a_2 - a_1}{12} = \frac{1}{24}.$$

$$\boxed{\therefore y(x) \approx 1 - \frac{1}{2} x^2 + \frac{1}{24} x^4}$$

$$y'' + 5y' + 4y = 2\delta(t-5),\ y(0)=0,\ y'(0)=0$$

$$\Rightarrow s^2 Y + 5sY + 4Y = 2e^{-5s}.$$

$$(s^2+5s+4)Y = 2e^{-5s}$$

$$Y(s) = \frac{2}{s^2+5s+4} e^{-5s}.$$

$$\frac{2}{s^2+5s+4} = \frac{A}{s+4} + \frac{B}{s+1} \Rightarrow 2 = A(s+1) + B(s+4)$$

$\underline{s=-4}$: $A = \frac{-2}{3}$; $s=-1$: $B = 2/3$

$$\therefore \frac{2}{s^2+5s+4} = \frac{-2/3}{s+4} + \frac{2/3}{s+1}$$

$$\mathcal{L}^{-1}\left(\frac{2}{s^2+5s+4}\right) = -\tfrac{2}{3}e^{-4t} + \tfrac{2}{3}e^{-t}, \text{ so}$$

$$\boxed{y(t) = \left[-\tfrac{2}{3}e^{-4(t-5)} + \tfrac{2}{3}e^{-(t-5)}\right]u(t-5)}$$