Math. 350-05
Fall, 2017
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Final Exam

Tuesday, December 5, 2017
11:00AM to 1:30PM

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Problems 1 and 2 are each worth 30 points, and problems 3 and 4 are each worth 20 points.

1. Consider $y' + 9y = 18$.

   (a) What is the equilibrium solution of this equation?

   (b) Is the equilibrium stable, unstable, or neither? Why?

   (c) Solve the initial value problem $y' + 9y = 18$, $y(0) = 1$.

   (d) Does the solution of the above initial value problem approach the equilibrium solution? If so, for what value of the independent variable does the solution become within 10% of the equilibrium?

2. In a spring-mass-damper system, A mass of 5 kilograms is hung from a spring, with a damper attached. In a separate experiment, it takes 2.5 Newtons to stretch the spring 10 centimeters. When the damper was tested, 10 Newtons of force was measured when the damper was moving 1 meter per second. The weight is initially moved to 1 meter from its equilibrium position in the positive direction, and a velocity of 1 meter per second in the positive direction is imparted to it. After that, no external forces are applied to the system.

   (a) Write down the initial value problem corresponding to the system.

   (b) Solve the initial value problem.

   Be sure to arrange your steps in a logical and orderly manner.

3. Solve the following initial value problem:

$$y'' + 9y = \sin(t) - u_{2\pi}(t)\sin(t - 2\pi); \quad y(0) = 0, \quad y'(0) = 0.$$

   (You may use the supplied table of Laplace transforms.)

4. Solve the following initial value problem:

$$y'' + 9y = 1 - \delta(t - 2\pi); \quad y(0) = 0, \quad y'(0) = 0.$$

   (You may use the supplied table of Laplace transforms.)

Table 1: Table of Laplace Transforms

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. t^n, $n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28