Math. 350-05
Fall, 2017
R. B. Kearfott

First Exam

Wednesday, September 20, 2017

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 25 points.

1. Consider $y' + 2y = 4$.

   (a) Does this equation have an equilibrium solution? If so, what is it?

   (b) If there is an equilibrium solution, is it stable, unstable, or neither? State why.

   (c) Sketch a direction field for this equation, showing any equilibrium solutions.

2. A certain small bay contains 83×10^6 cubic feet of water, and normally has a salinity of about 1.6%, corresponding to a concentration of about 1 pound of salt per cubic foot. In an exceptionally dry year, no water flowed into the bay over the summer, and the salinity rose to that of the open ocean, namely, about 3.5%, or about 2.2 pounds per cubic foot. With a return of rains, the river started discharging fresh water (0 pounds salt per cubic foot) at the rate of 10.4×10^6 cubic feet per day, and the fresh water mixes with the bay water. The bay water then flows into the ocean at the same rate. Biologists have determined that certain brackish-water fish will die unless the salinity returns to 2.5% (less than about 1.5 pounds of salt per cubic foot) within a week.

   (a) Write down an initial value problem that models the total amount of salt $S(t)$ in the bay at time t days after the river started flowing.

   (b) Solve the initial value problem.

   (c) Compute the total amount of salt in the bay at time $t = 7$.

   (d) Will the salinity in the bay be at 2.5% or less at time $t = 7$?

3. Solve the following initial value problem: $\quad y' - \dfrac{3}{t}y = t^3, \quad y(1) = 1.$

4. Solve the following initial value problem:

$$\frac{dy}{dx} = \sin(x)e^y, \quad y(0) = 0.$$