

Second Exam

Wednesday, October 18, 2017

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each part of each problem is worth 16 points, and 4 points are free.

1. Find solutions to the following initial value problems.

(a) $y'' + 2y' + 5y = 0$, $y(0) = 1$, $y'(0) = 1$.

(b) $y'' + 6y' + 9y = 0$, $y(0) = 1$, $y'(0) = -5$.

2. Find the general solution to the following differential equation:

$$y'' + 25y = \sin(5t).$$

3. Write $\cos(10^6 t) + \sqrt{3} \sin(10^6 t)$ in the form $R \cos(\omega_0 t - \delta)$. After doing that, identify the amplitude, the phase shift, and the angular velocity (also known as the natural frequency).

4. In a spring-mass-damper system, A mass of 5 kilograms is hung from a spring, with a damper attached. In a separate experiment, it takes 2.5 Newtons to stretch the spring 10 centimeters. When the damper was tested, 10 Newtons of force was measured when the damper was moving 1 meter per second. The weight is initially moved to 1 meter from its equilibrium position in the positive direction, and a velocity of 1 meter per second in the positive direction is imparted to it. After that, no external forces are applied to the system.

(a) Write down the initial value problem corresponding to the system.

(b) Write down the solution to that initial value problem.

Be sure to arrange your steps in a logical and orderly manner.