1 a The equilibrium is where $9y=18$, i.e. $y = 18/9 = \boxed{2}$.

b The equilibrium is stable since $y' = -9y+18$, so $y'<0$ for $y>2$ and $y'>0$ for $y<2$.

c $\frac{dy}{dt} = 18-9y = -9(y-2)$, giving

$\int \frac{dy}{y-2} = \int -9\,dt$, so $\ln|y-2| = -9t+C$,

so $y(t) = 2+Ke^{-9t}$ $y(0)=1=2+K \Rightarrow K=-1$,

so $y(t) = 2-e^{-9t}$

d Yes, the solution tends to the equilibrium. The value T of t for which the solution becomes within 10% of the equilibrium obeys:

$|\cancel{2}-e^{-9t}| \leq (0.1)(2)$, i.e. $e^{-9t} \leq 0.2$.

$\Leftrightarrow t \leq -\log_e(0.2)/9 \cong .1789$

2) $my'' + \gamma y' + ky = f(t)$, where m is the mass, γ is the damping coefficient, and k is the spring constant. We will use the MKS system for this problem (Meters-Kilograms-Seconds).

We have $k = 2.5/0.1 = 25$ Newtons/meter.

$\gamma = 10/1 = 10$ Newtons/(meter/second)

$m = 5$ kilograms.

a) $\boxed{5y'' + 10y' + 25y = 0, \ y(0) = 1, \ y'(0) = 1.}$

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b) $y'' + 2y' + 5y = 0$

$r^2 + 2r + 5 = 0 \Rightarrow r = \frac{-2}{2} \pm \frac{\sqrt{4 - 4(1)(5)}}{2} = \boxed{-1 \pm 2i}$.

Thus, the general solution is

$y(t) = C_1 e^{-t}\cos(2t) + C_2 e^{-t}\sin(2t)$.

$y(0) = \boxed{1 = C_1}$

$y'(t) = -C_1 e^{-t}\cos(t) - 2C_1 e^{-t}\sin(2t)$

$\quad + -C_2 e^{-t}\sin(2t) + 2C_2 e^{-t}\cos(2t)$,

so $y'(0) = -C_1 + 2C_2 = 1 \Rightarrow \boxed{C_2 = 1}$.

Thus, $\boxed{y(t) = e^{-t}(\cos(2t) + \sin(2t))}$.

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3) The Laplace transform of the IVP is:

$$s^2 Y + 9Y = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}$$

$$\Rightarrow Y(s) = \frac{1}{(s^2+9)(s^2+1)}\left[1 - e^{-2\pi s}\right].$$

$$\frac{1}{(s^2+9)(s^2+1)} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+1} \Rightarrow (As+B)(s^2+1) + (Cs+D)(s^2+9) = 1$$

$$\Rightarrow 1 = (A+C)s^3 + (B+D)s^2 + (A+9C)s + (B+9D)$$

$$\Rightarrow \left\{\begin{matrix} A+C=0 \\ A+9C=0 \end{matrix}\right\}, \left\{\begin{matrix} B+D=0 \\ B+9D=1 \end{matrix}\right\} \Rightarrow A=0, C=0, D=\frac{1}{8}, B=\frac{-1}{8} \Rightarrow$$

$$Y(s) = \left[\frac{-\frac{1}{8}}{s^2+9} + \frac{\frac{1}{8}}{s^2+1}\right]\left[1-e^{-2\pi s}\right] = \left[\frac{-1}{24}\frac{3}{s^2+9} + \frac{1}{8}\frac{1}{s^2+1}\right]\left[1-e^{-2\pi s}\right]$$

Thus,
$$\boxed{\begin{aligned} y(t) &= \frac{-1}{24}\sin(3t) + \frac{1}{8}\sin(t) \\ &\quad - u_{2\pi}(t)\left[\frac{-1}{24}\sin(3t) + \frac{1}{8}\sin(t)\right] \end{aligned}}$$

4 The Laplace transform of the IVP is:

$$s^2Y + 9Y = \frac{1}{s} - e^{-2\pi s}, \text{ whence}$$

$$Y(s) = \frac{1}{s(s^2+9)} - \frac{e^{-2\pi s}}{s^2+9}.$$

$$\frac{1}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9} \Rightarrow A(s^2+9) + (Bs+C)s = 1$$

$$\Rightarrow (A+B)s^2 + Cs + A = 1 \Rightarrow C = 0, A = \frac{1}{9}, B = \frac{-1}{9}$$

$$\Rightarrow Y(s) = \frac{1/9}{s} - \frac{s(1/9)}{s^2+9} - \frac{e^{-2\pi s}}{s^2+9} = \frac{1}{9}\left(\frac{1}{s}\right) - \frac{1}{9}\left(\frac{s}{s^2+9}\right) + e^{-2\pi s}\left(\frac{-1}{3}\,\frac{3}{s^2+9}\right)$$

whence $\boxed{y(t) = \frac{1}{9} - \frac{1}{9}\cos(3t) - \frac{1}{3}u_{2\pi}(t)\sin(3(t-2\pi))}$