1 a The characteristic equation is:

$$r^2+2r+5=0, \text{ so } r=\frac{-2\pm\sqrt{4-20}}{2}=-1\pm 2i.$$

Thus, the homogeneous part of the solution is:

$$y(t)=C_1e^{-t}\cos 2t+C_2e^{-t}\sin(2t)$$
$$=e^{-t}(C_1\cos 2t+C_2\sin(2t)).$$

Since there is no forcing term, this is the general solution.

$$y(0)=\boxed{C_1=1}.$$

$$y'(t)=-e^{-t}(C_1\cos 2t+C_2\sin(2t))$$
$$+e^{-t}(-C_2\sin(2t)+2C_2\cos(2t))$$

so

$$y'(0)=-C_1+2C_2=-1+2C_2=1\Rightarrow C_2=1.$$

$$\boxed{\therefore y(t)=e^{-t}(\cos(2t)+\sin(2t))}$$

---

b $r^2+6r+9=(r+3)^2=0\Rightarrow r=-3.$

Since r is a double root, the general solution is $y(t)=c_1e^{-3t}+c_2te^{-3t}$.

$$y(0)=c_1=1.$$

$$y'(t)=-3c_1e^{-3t}+c_2e^{-3t}-3c_2te^{-3t},$$

so $y'(0)=-3c_1+c_2=-3+c_2=-5.$

Thus, $c_2=-2$, and the general solution to the IVP is

$$\boxed{\therefore y(t)=e^{-3t}-2te^{-3t}}$$

---

(2) $r^2+25=0 \Rightarrow r=\pm 5i$, so the solution to the homogeneous part is:

$$y_h(t) = C_1\cos(5t)+C_2\sin(5t).$$

Since the forcing term is also a solution to the homogeneous equation, we assume a solution of the form $y_p(t) = t(A\cos 5t + B\sin(5t))$

$$y_p' = A\cos 5t + B\sin 5t + t(-5A\sin 5t + 5B\cos 5t)$$

$$y_p'' = -5A\sin 5t + 5B\cos 5t - 5A\sin 5t + 5B\cos 5t + t(-25A\cos 5t - 25B\sin(5t))$$

$$= -10A\sin(5t) + 10B\cos(5t) + t(-25A\cos 5t - 25B\sin(5t))$$

Thus, $y_p''+25y_p = -10A\sin(5t)+10B\cos 5t = \sin(5t)$

$$\Rightarrow A = -\frac{1}{10},\ B=0.$$

Thus, the general solution is:

$$\boxed{y(t) = C_1\cos 5t + C_2\sin(5t) - \frac{1}{10}t\cos(5t)}$$

---

(3) $R^2 = (1)^2+(\sqrt{3})^2 = 1+3=4 \Rightarrow \boxed{R=2}$

$\cos(\delta) = \frac{1}{2}$, $\sin(\delta) = \frac{\sqrt{3}}{2} \Rightarrow \delta = \frac{\pi}{3}$.

Thus $\cos(10^6 t) + \sqrt{3}\sin(10^6 t) = \boxed{2\cos\left(10^6 t - \frac{\pi}{3}\right)}$

The amplitude is 2, the phase shift is $\frac{\pi}{3}$, and the angular velocity is 10^6.

---

4 The MKS system will be used.

a $mu'' + \gamma u' + ku = 0;\ u(0) = 1, u'(0) = 1.$

$m = \cancel{.5}5,\ \gamma = \frac{10}{1},\ k = \frac{2.5}{0.1} = 25,$

so the equation is

$$5u'' + 10u' + 25u = 0, \text{ with } u(0) = 1, u'(0) = 1.$$

Note: Due to errors and inconsistencies in the problem statement, this problem was not graded.

b ~~The~~ Dividing the equation by 5 gives

$$u'' + 2u' + 5u = 0, u(0) = 1, u'(0) = 1.$$

This is the same IVP as in problem 1(a), so its solution is:

$$\boxed{u(t) = e^{-t}[\cos(2t) + \sin(2t)]}$$