Math. 350-02
Spring, 2009
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Final Exam

Friday, May 8, 2009, 2:00PM to 4:30PM

This exam is closed book. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. The problem will be graded both on correctness and on how logical your presentation is. Each entire problem is worth 20 points. You may keep this sheet after turning in your exam.

1. Classify each of the following differential equations as linear or non-linear. In the case of the nonlinear equations, state why.

   (a) $\left(\dfrac{dy}{dt}\right)^2 + e^t y = t.$ (b) $y' + e^t y = 0.$

   (c) $y''' + t^2 y' + y = e^{-t}.$ (d) $t\sin(y') + y = e^{-t}.$

2. A pond of total volume 10,000 cubic meters has a flow entering it of 10,000 cubic meters per year. There is a pollutant entering the pond, whose concentration in grams per cubic meter is decreasing as a function of t years, according to

   $$c_{\text{in}}(t) = 100(1 - t/10),$$

   so that, after 10 years, the water entering the pond is pure. Assume that water is flowing out of the pond at the same rate that it is flowing in, and that the initial concentration in the pond is 100 grams per cubic meter.

   (a) Find the concentration $c(t)$ of pollutants in the pond as a function of time t.

   (b) What is the concentration after 10 years?

3. Write

   $$\frac{e^{(-4+3i)t} - e^{(-4-3i)t}}{2i}$$

   in the form $a(t) + b(t)i$.

4. Solve the initial value problem

   $$y'' + 4y' + 4y = \sin(2t), \quad y(0) = 0, \quad y'(0) = 0.$$

   by writing down the characteristic equation and finding its solutions.

5. Find terms, up to and including degree 5, of the series solution to the initial value problem

   $$y'' + xy' + y = 1, \quad y(0) = 3, \quad y'(0) = 4.$$