Second Exam
Friday, February 13, 2009

This exam is closed book. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 32 points, and 4 points are "free."

1. Solve the following initial value problem. (Show all steps.)

$$\frac{dy}{dt} + \frac{1}{t}y = \frac{1}{t}, \quad y(1) = 0.$$

2. Consider the initial value problem from Problem 1.

   (a) Do one step of Euler's method with $h = 0.1$, starting at $t = 1$, $y = 0$. Carry 4 significant digits in your computations.

   (b) Do two steps of Euler's method with $h = 0.05$, starting at $t = 1$, $y = 0$. Carry at least 4 significant digits in your computations.

   (c) Compare the values obtained from the previous two parts of this problem to the value $y(1.1)$ you obtain by plugging 1.1 into the exact solution from Problem 1. Which of the values is closer to the exact value?

3. Suppose runoff from a farm enters a stream and pollutes it with 5 parts per million of an insecticide that does not degrade in the environment. Suppose the stream's flow per year is 5×10^5 units, and suppose the stream flows into a lake with volume 10^6 units. Suppose the lake initially has no insecticide in it, and a stream flows out of the lake at 5×10^5 units per year.

   (a) Write down and solve a differential equation for the total amount $A(t)$ of insecticide in the lake at time t years.

   (b) After how long will the insecticide in the lake reach 75% of its equilibrium value?