① $\mu(t) = e^{\int \frac{dt}{t}} = e^{\ln(t)} = t$

$\frac{d(ty)}{dt} = 1 \Rightarrow ty = t + k \Rightarrow y = 1 + \frac{k}{t}$.

$y(1) = 0 = 1 + \frac{k}{1} \Rightarrow k = -1 \Rightarrow \boxed{y(t) = 1 - \frac{1}{t}}$.

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② $y(0.1) \approx y(0) + (0.1)y'(1)$. $y_{k+1} = y_k + h f(t_k, y_k)$

(a) $y' = \frac{-1}{t}y + \frac{1}{t} = f(t,y)$.

$y_0 = 0, t_0 = 1$.

$y_1 = 0 + \left(\frac{0}{1} \cdot 0 + \frac{1}{1}\right)(0.1) = +0.1 \approx y(1.1)$

(b) $y_0 = 0, t_0 = 1$.

$y_1 = 0 + 0.05\left(\frac{1}{1}(0) - \frac{1}{1}\right) = +0.05 \approx y(1.05)$

$y_2 = -0.05 + 0.05\left(\frac{-1}{1.1}(-0.05) + \frac{1}{1.1}\right) \approx +0.09548 \approx y(1.1)$

(c) Here are the values and errors:

$y(1.1) = 1 - \frac{1}{1.1}$

	$h = 0.1$	$h = 0.05$	exact
value	+0.1000	+.09548	0.090909
error	9.09×10^{-3}	6.8×10^{-3}	— 0 —

The $h = 0.05$ solution is closer.

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③ Check the solutions on page ② of the answers for the Fall, 2004 Math. 350, first exam.