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$$\mathcal{L}(y''+3y'+2y) = \mathcal{L}(u_3(t)e^{-(t-3)})$$

Using formula 13 and formula 2.

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = e^{-3s}\cdot\frac{1}{s+1}$$

$$(s^2+3s+2)Y - 1 = e^{-3s}\cdot\frac{1}{s+1}$$

$$Y(s) = \frac{1}{(s+1)(s+2)} + \frac{1}{(s+1)(s+2)}\frac{e^{-3s}}{(s+1)}.$$

Using a partial fraction decomposition:

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \qquad 1 = A(s+2) + B(s+1) \Rightarrow \begin{array}{l} A+B=0 \\ 2A+B=1 \\ \hline A=1,\ B=-1 \end{array}$$

$$\frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}.$$ Thus,

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2} + \frac{e^{-3s}}{(s+1)^2} - \frac{e^{-3s}}{(s+1)(s+2)}$$

$$= \frac{1}{s+1} - \frac{1}{s+2} + \frac{e^{-3s}}{(s+1)^2} - \frac{e^{-3s}}{s+1} + \frac{e^{-3s}}{s+2}.$$

This gives

$$\boxed{y(t) = e^{-t} - e^{-2t} + u_3(t)\left[(t-3)e^{-(t-3)} - e^{-(t-3)} + e^{-2(t-3)}\right]}$$

formulas 11 and 13

formulas 2 and 13.