1 (a) This is nonlinear, because the derivative is squared.

(b) Linear.

(c) Linear

(d) This is nonlinear, since the derivative is an argument to a nonlinear function.

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2 See the answer to exam 2 from Fall, 2007, problem 2

3
$$e^{(-4+3i)t} = e^{-4t}\cos 3t + i e^{-4t}\sin 3t$$
$$-\left[e^{(-4-3i)t} = e^{-4t}\cos 3t - i e^{-4t}\sin 3t\right]$$
$$\div 2i \quad e^{(-4+3i)t} - e^{(-4-3i)t} = 2i e^{-4t}\sin 3t$$
$$\frac{e^{(-4+3i)t} - e^{(-4-3i)t}}{2i} = \boxed{e^{-4t}\sin(3t)}$$

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4 $r^2+4r+4=(r+2)^2=0$. $r=-2$ is a double root, so the general solution to the homogeneous equation is $\boxed{y_h(t) = C_1 e^{-2t} + C_2 t e^{-2t}}$

A particular solution to the nonhomogeneous equation is of the form: $y_p(t) = A\cos 2t + B\sin 2t$.
$$y_p' = -2A\sin 2t + 2B\cos 2t$$
$$y_p'' = -4A\cos 2t - 4B\sin 2t$$
$$y_p'' + 4y_p' + 4y_p = (-4A+8B+4A)\cos(2t) + (-4B-8A+4B)\sin(2t)$$
$$= 8B\cos(2t) - 8A\sin(2t) = \sin(2t)$$

Thus, $8B=0 \Rightarrow B=0$

$-8A = 1 \Rightarrow A = \frac{-1}{8}$, so $y_p(t) = \frac{-1}{8}\cos(2t)$

Thus
$$y(t) = y_p(t) + y_h(t)$$
$$y(0) = \boxed{\frac{-1}{8} + C_1 = 0} \Rightarrow \boxed{C_1 = \frac{1}{8}}$$
$$y'(t) = \frac{1}{4}\sin(2t) - 2C_1 e^{-2t} + C_2 e^{-2t} - 2C_2 t e^{-2t}$$
$$y'(0) = \boxed{-2C_1 + C_2 = 0} \Rightarrow C_2 = +2C_1 \quad \text{(continued)}$$
$$\Rightarrow \boxed{C_2 = \frac{1}{4}}$$

Thus, the solution is:

$$\boxed{y(t) = -\frac{1}{8}\cos(2t) + \frac{1}{8}e^{-2t} + \frac{1}{4}te^{-2t}.}$$

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(5) $y(x) = \sum_{n=0}^{\infty} a_n x^n$

$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \Rightarrow xy' = \sum_{n=1}^{\infty} n a_n x^n.$

$y'' = \sum_{n=2}^{\infty} n(n-1)x^{n-2}a_n = \sum_{m=0}^{\infty}(m+2)(m+1)x^m a_{m+2}$ $\leftarrow$ $\boxed{\begin{array}{l} m = n-2 \\ n = m+2 \end{array}}$

So $y'' + xy' + y = a_0 + 2a_2 + \sum_{n=1}^{\infty}\left[a_n + na_n + (n+2)(n+1)a_{n+2}\right]x^n = 1.$

This gives: $a_0 + 2a_2 = 1 \Rightarrow a_2 = \frac{1-a_0}{2}$

and $(n+1)a_n + (n+2)(n+1)a_{n+2} = 0, \quad n \geq 1$

$\Rightarrow a_{n+2} = -\frac{a_n}{n+2} \quad n \geq 1.$

The initial conditions give $\boxed{a_0 = 3}, \boxed{a_1 = 4,}$

$a_2 = \frac{1-a_0}{2} = \frac{1-3}{2} = \boxed{-1 = a_2}$

$a_3 = \frac{-a_1}{3} = \frac{-4}{3}$

$a_4 = \frac{-a_2}{4} = \frac{1}{4}$

$a_5 = \frac{-a_3}{5} = \frac{-4/3}{5} = \frac{+4}{15}$

$$\boxed{\therefore\ y(x) \approx 3 + 4x - x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + \frac{4}{15}x^5}$$