Math. 350-02
Spring, 2016
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First Exam

Thursday, February 11, 2016

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 25 points.

1. At time $t = 0$, a large parachute deploys on a falling object that is falling at the rate of 49 meters per second. Once the parachute deploys, the velocity v of the object is governed by the differential equation

$$\frac{dv}{dt} = 9.8 - 19.6v$$

   (a) Find the velocity v of the object as a function of time t, for $t \geq 0$.

   (b) Approximately how fast is the object falling after half a second?

   (c) Approximately how fast will the object be falling after a long period of time?

2. Classify each of the following differential equations as linear or nonlinear, and state why in each case in which it is nonlinear.

   (a) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + 2e^t y = t.$ (b) $y' + te^y = 0.$

   (c) $y''' + 3y'' + y = e^{-t}.$ (d) $\dfrac{dy}{dx} = x^2y^2.$

3. Find the solution to the initial value problem

$$\frac{dy}{dt} - 2ty = t, \quad y(0) = 1.$$

4. A tank with a capacity of 500 gallons is initially filled with water containing 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, while the mixture is flowing out of the tank at the same rate. Let $Q(t)$ be the amount of salt in the tank.

   (a) Find the amount of salt in the tank at any time t.

   (b) Approximately what amount of salt will be in the tank after a long period of time?

   (c) Approximately how much time will pass for the salt to reach 50% of its limiting value?