[illegible] 350-02, Spring, 2016 First exam answers, page 1

1) $\dfrac{dv}{dt} = -19.6(v - \tfrac{1}{2})$

$\displaystyle\int \frac{dv}{v - 1/2} = \int -19.6\,dt.\quad \ln|v - \tfrac{1}{2}| = -19.6t + k \Rightarrow v = \tfrac{1}{2} + Ce^{-19.6t}$

$v(0) = 49 \Rightarrow \tfrac{1}{2} + C = 49 \Rightarrow C = +48.5$

$\boxed{(a)\ v(t) = \tfrac{1}{2} + 48.5e^{-19.6t}}$

(b) $v(1/2) = \tfrac{1}{2} + 48.5e^{-9.8} \approx$ $\boxed{.502 \text{ meters per second}}$

$\boxed{(c)\ .5 \text{ meters per second}}$

2) (a) Linear, second-order

(b) Nonlinear because of the e^y term, first order.

(c) Linear, third order.

(d) Nonlinear because of the y^2 term, first order

3) An integrating factor should have $u' = -2tu$

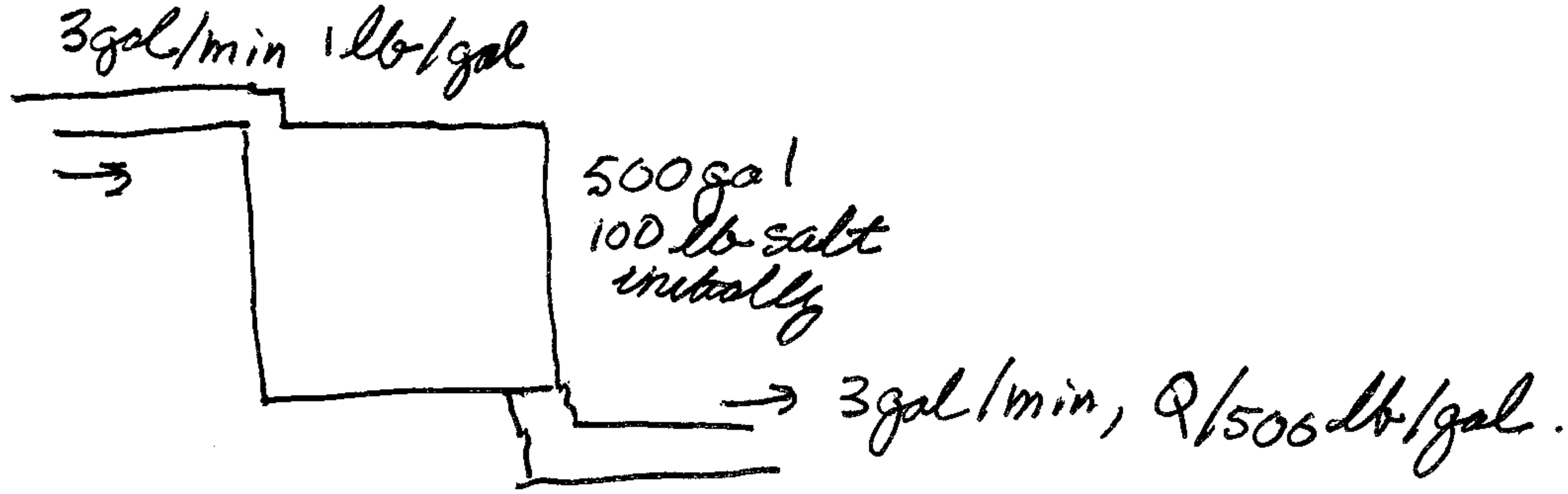
$= \ln|u| = -t^2 \Rightarrow u = e^{-t^2}$ will do.

$(e^{-t^2}y)' = te^{-t^2} \Rightarrow e^{-t^2}y = -\tfrac{1}{2}e^{-t^2} + C$

$\Rightarrow \boxed{y(t) = -\tfrac{1}{2} + Ce^{t^2}}$. $y(0) = C - \tfrac{1}{2} = 1 \Rightarrow C = 3/2$

$\boxed{\cancel{\therefore y(t) = -\tfrac{1}{2} + \tfrac{3}{2}e^{t^2}}}$ $\boxed{\therefore y(t) = -\tfrac{1}{2} + \tfrac{3}{2}e^{t^2}}$

(4) (a)



$$\frac{dQ}{dt} = 3 - \frac{3Q}{500}, \; Q(0)=100; \quad \frac{dQ}{dt} = \frac{-3}{500}(Q-500)$$

$$\int \frac{dQ}{Q-500} = \int \frac{-3}{500}\,dt. \Rightarrow \ln|Q-500| = \frac{-3t}{500} + k$$

$$\Rightarrow Q(t) = 500 + Ce^{-\frac{3t}{500}}. \quad Q(0) = 100 = 500 + C \Rightarrow C = -400.$$

$$\boxed{\therefore Q(t) = 500 - 400e^{-\frac{3t}{500}}}$$

(b) The limiting value is 500 lb.

(c) $250 = 500 - 400e^{-\frac{3T}{500}} \Rightarrow \frac{-250}{-400} = e^{-\frac{3t}{500}}$

$$\Rightarrow \frac{3t}{500} = \ln\left(\frac{400}{250}\right) \Rightarrow t = \frac{500}{3}\ln\left(\frac{400}{250}\right) \cancel{\approx 15667\text{ min}}$$

$$\approx \boxed{78.3\text{ min.} \approx 1.3\text{ hours}} \; \cancel{\approx 9.4\text{ seconds}}$$